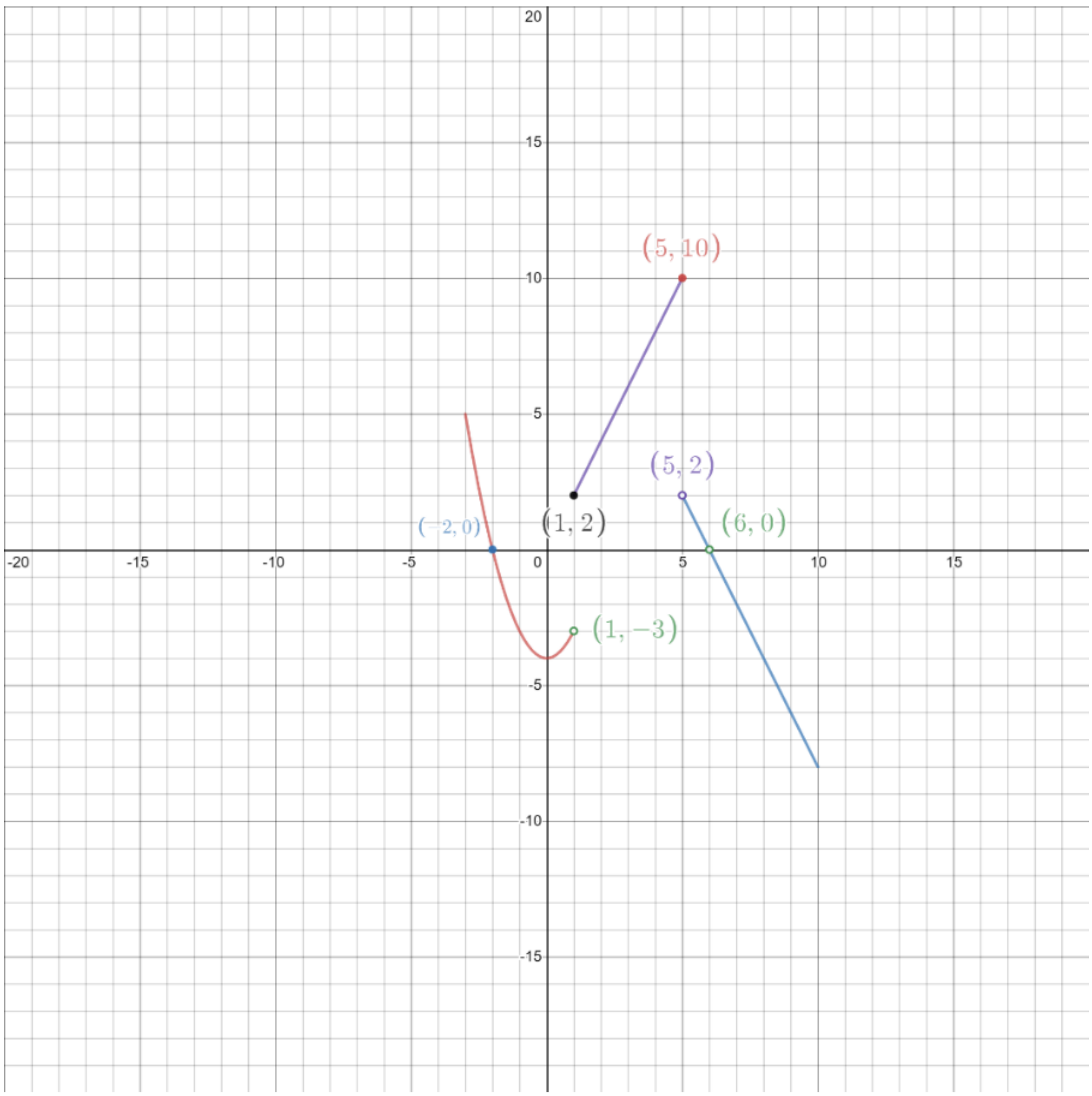
Section 1.1 Limits (minimum homework: 1.1 1-11 odds, 15, 19 and 21)

There is just a bit of review that is needed for this section.

Let us do that review now.

Use the graph of $f(x)$ given below to find the following:

$f(-2)$ $f(1)$ $f(5)$ $f(6)$



$f(-2) = 0 \quad the\ y - coordinate\ of\ the\ point\ (-2,0)$

$f(1) = 2 \quad the\ y - coordinate\ of\ the\ point\ (1,2),$

$the\ point\ with\ a\ solid\ circle\ above\ x = 1$

$f(5) = 10$

$f(6) \quad = undefined,$
$there\ is\ no\ point\ marked\ with\ a\ solid\ circle\ above\ or\ below\ x = 6$

Limit:

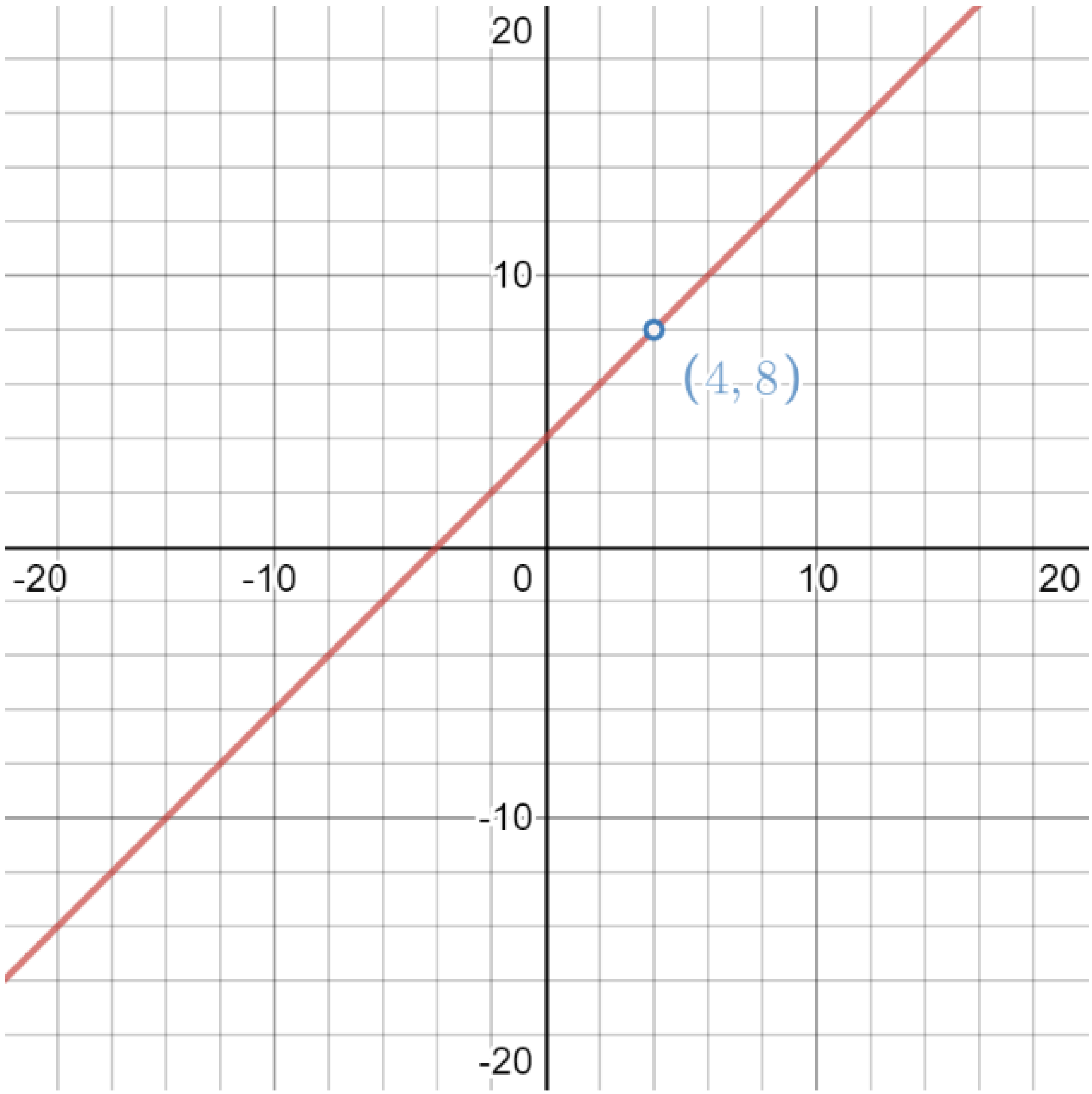
- The LIMIT of a function is the y-value that a function gets closer to as x approaches some given number.
- Limits describe how a function behaves near a point, instead of at that point.

Let us start by looking at the function: $f(x) = \frac{x^2-16}{x-4}$

We should notice that $f(4) = \frac{4^2-16}{4-4} = \frac{0}{0} = undefined$

- The function $f(x) = \frac{x^2-16}{x-4}$ is NOT defined when x = 4.
- x = 4 is not in the domain of $f(x) = \frac{x^2-16}{x-4}$.
- We can still talk about the limit of this function at x = 4, even though 4 is not in the domain of $f(x) = \frac{x^2-16}{x-4}$.
- The limit of $f(x) = \frac{x^2-16}{x-4}$ as x approaches 4 is a y-value as x gets infinitely near to 4.
- Limits describe how a function behaves near a point, instead of at that point.

Here is graph of $f(x) = \frac{x^2-16}{x-4}$ (notice the hole at the point $(4,8)$) This hole happens because the function is not defined at x = 4.



Let me try to show you how to understand limits at a specific value of x graphically.

I will add the appropriate Calculus symbols so we can start to get comfortable with them.

Each of these three statements are asking me to find the same value of y.

- Find the y-value that the function $f(x) = \frac{x^2-16}{x-4}$ approaches as x gets closer to x = 4.

- $\lim_{x \to 4} \frac{x^2-16}{x-4}$

- $\lim_{x \to 4} f(x)$

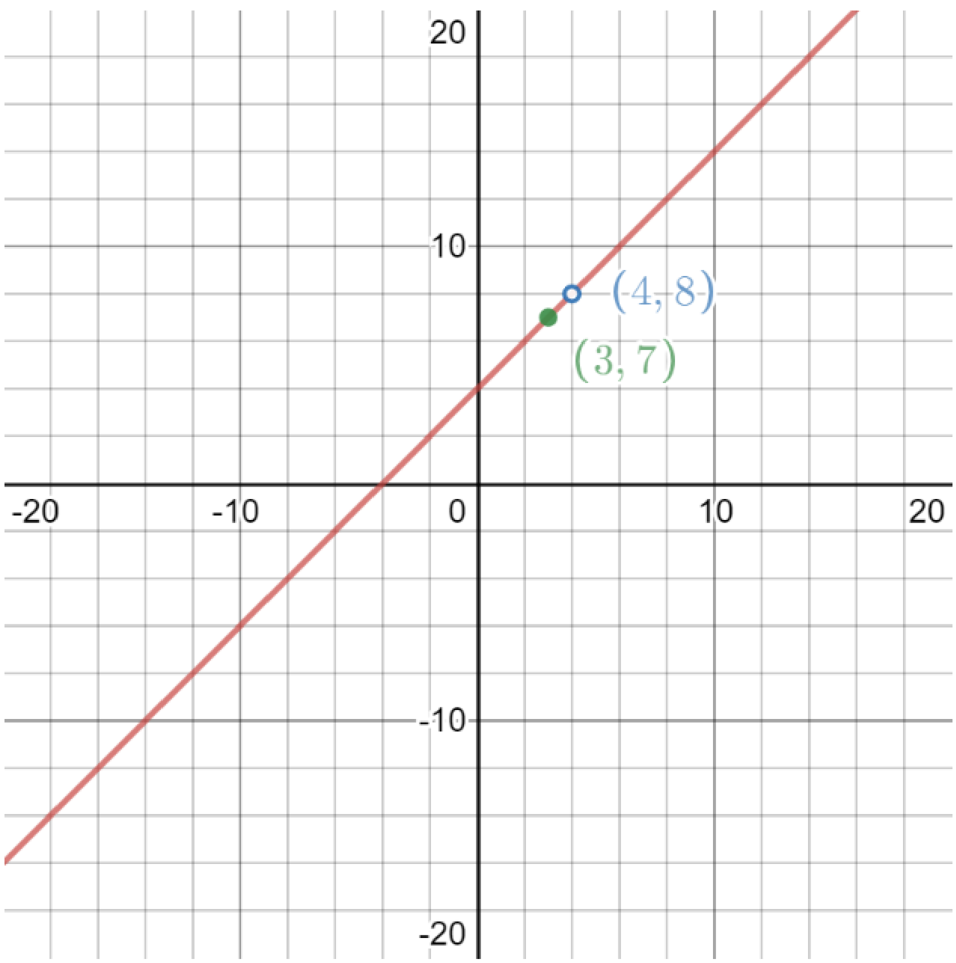
Let us look at the graph of $f(x) = \frac{x^2-16}{x-4}$ and examine points where "x" is close to 4. We will start with values of x that are less than 4.

My goal is to figure what happens to the y-value of points as x gets closer and closer to $x = 4$.

First: let x = 3 (3 is an arbitrary number I picked that is less than 4)

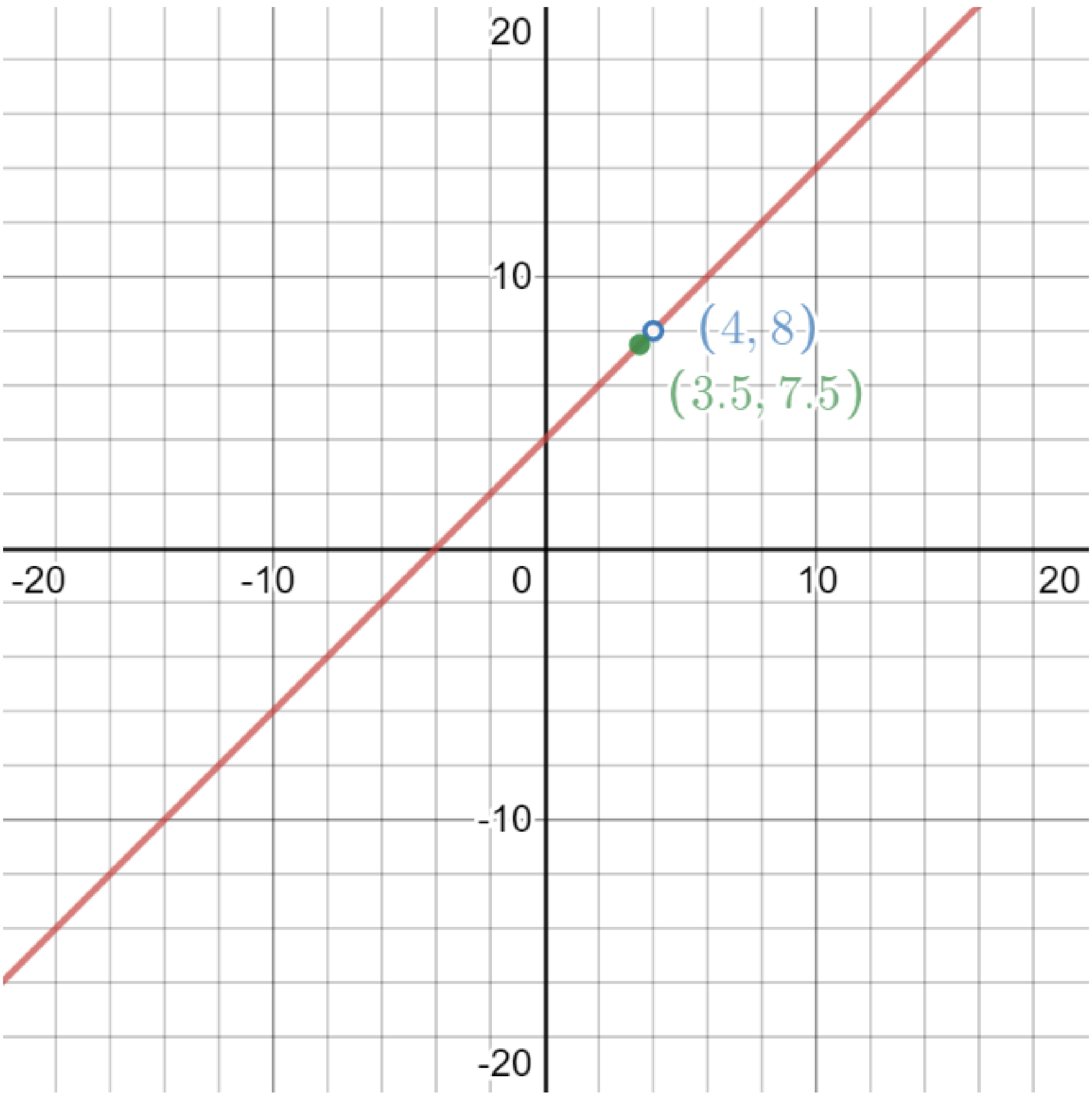
$f(3) = \frac{3^2-16}{3-4} = 7$ (this gives the point (3,7)

(you should notice that the point is not too far from the hole in graph)



Next: let x = 3.5 (3.5 is an arbitrary number I picked that is a bit closer to 4 then x = 3 was)

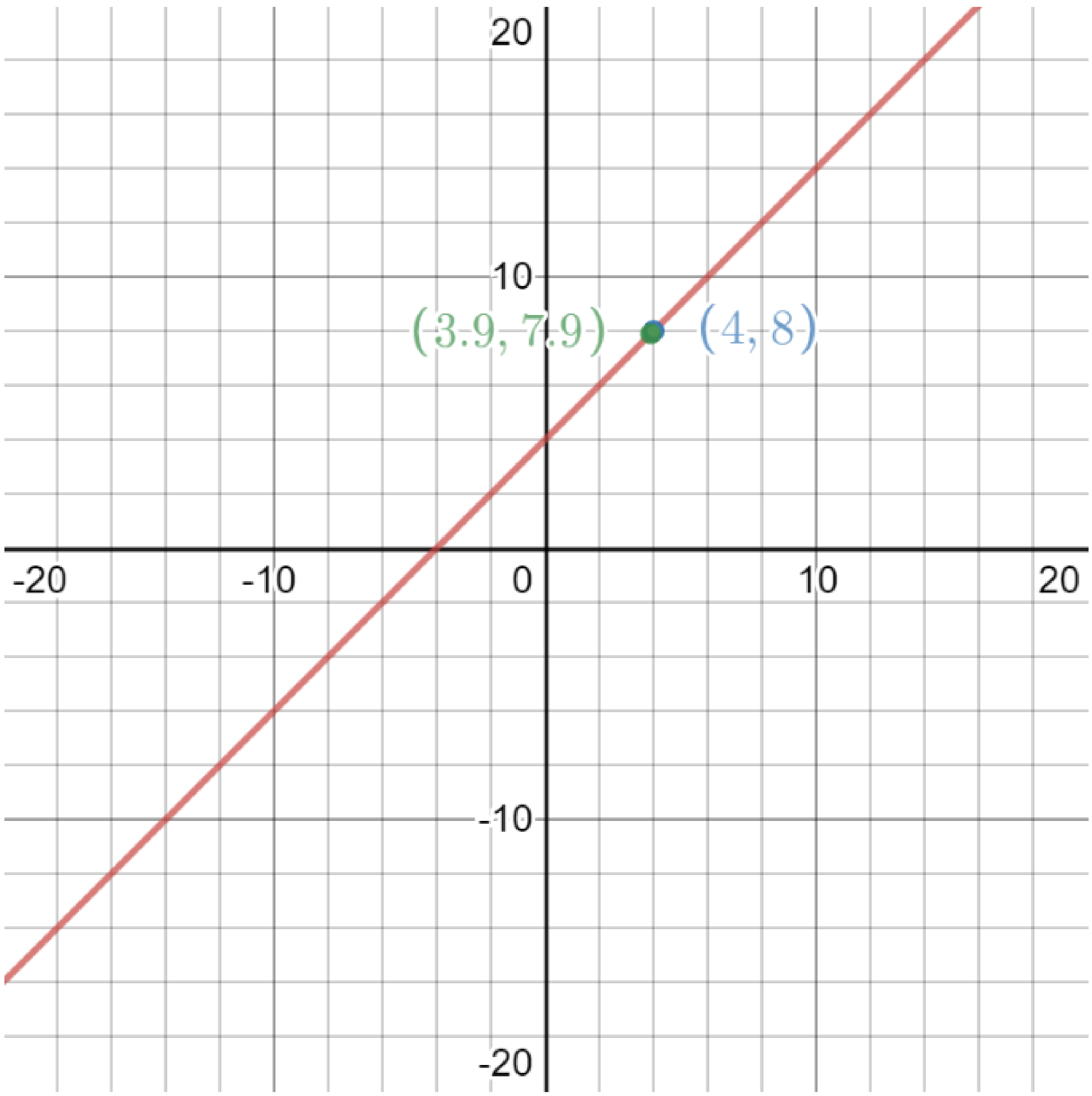
$f(3.5) = \frac{3.5^2-16}{3.5-4} = 7.5$ This creates the point (3.5, 7.5). You should notice that the point is even closer to the hole in the graph.



Next: let x = 3.9 (3.9 is an arbitrary number I picked that is a bit closer to 4 then x = 3.5 was)

$f(3.9) = \frac{3.9^2 - 16}{3.9 - 4} = 7.9$ (graphically this gives the point (3.9, 7.9)

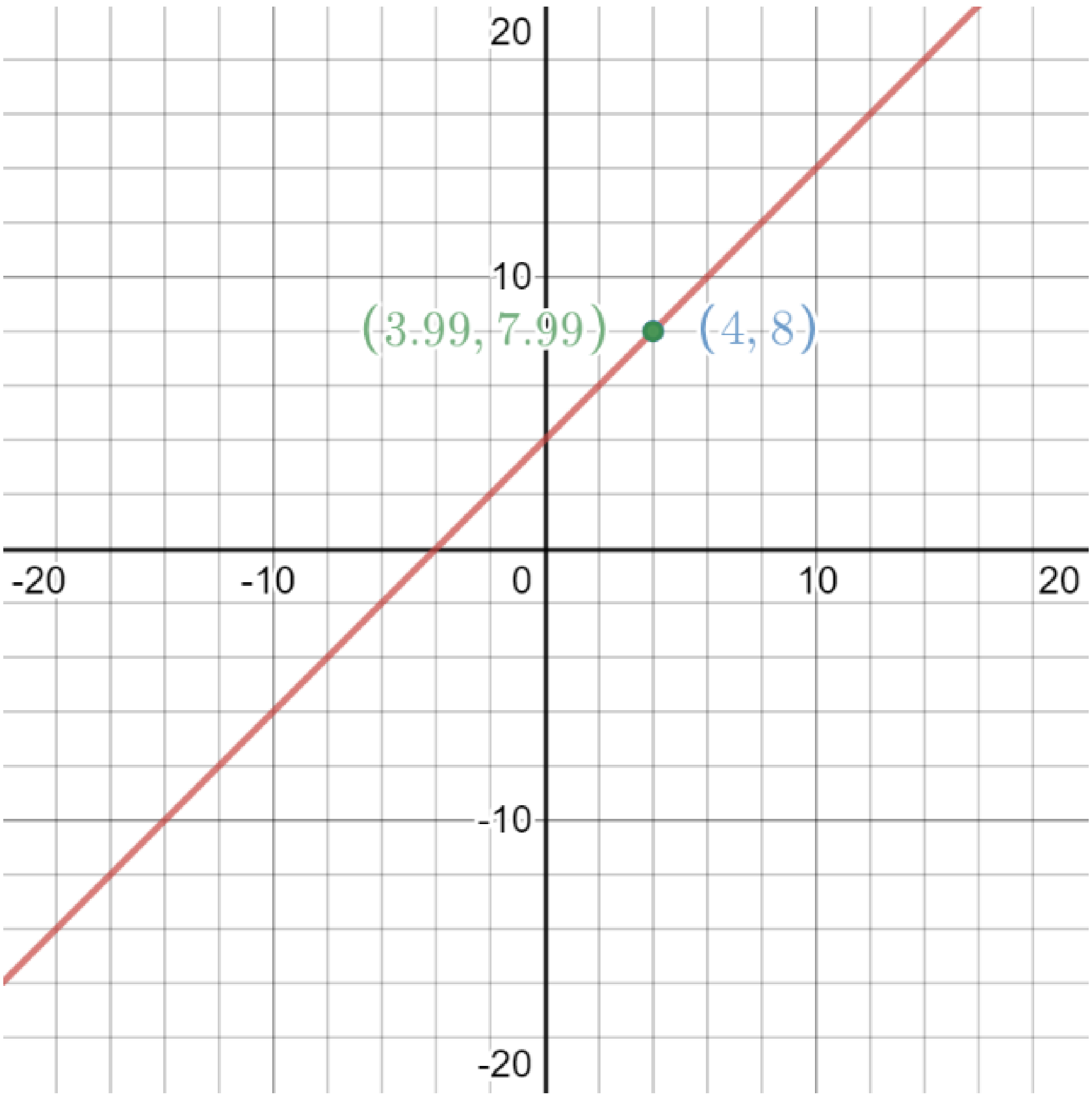
(you should notice that the point is even closer to the hole in the graph. In fact the point (3.9, 7.9) is so close to the hole that you cannot distinguish the two points.)



Next: let x = 3.99 (3.99 is an arbitrary number I picked that is a bit closer to 4 then x = 3.9 was)

$f(3.99) = \frac{3.99^2-16}{3.99-4} = 7.99$ (graphically this gives the point (3.99, 7.9))

(you should notice that the point is even closer to the hole in the graph. In fact the point (3.99, 7.99) is so close to the hole that you cannot distinguish the two points.)



Now I can make a conclusion about the y-value that my points are approaching as the x-values get closer to 4 **(but remain smaller than 4).**

Each of these statements are equivalent:

- As the values of x that are **smaller** than x = 4 get closer to x = 4 the y-values get closer to 8.

- $\lim_{x \to 4^-} \frac{x^2-16}{x-4} = 8$

- $\lim_{x \to 4^-} f(x) = 8$

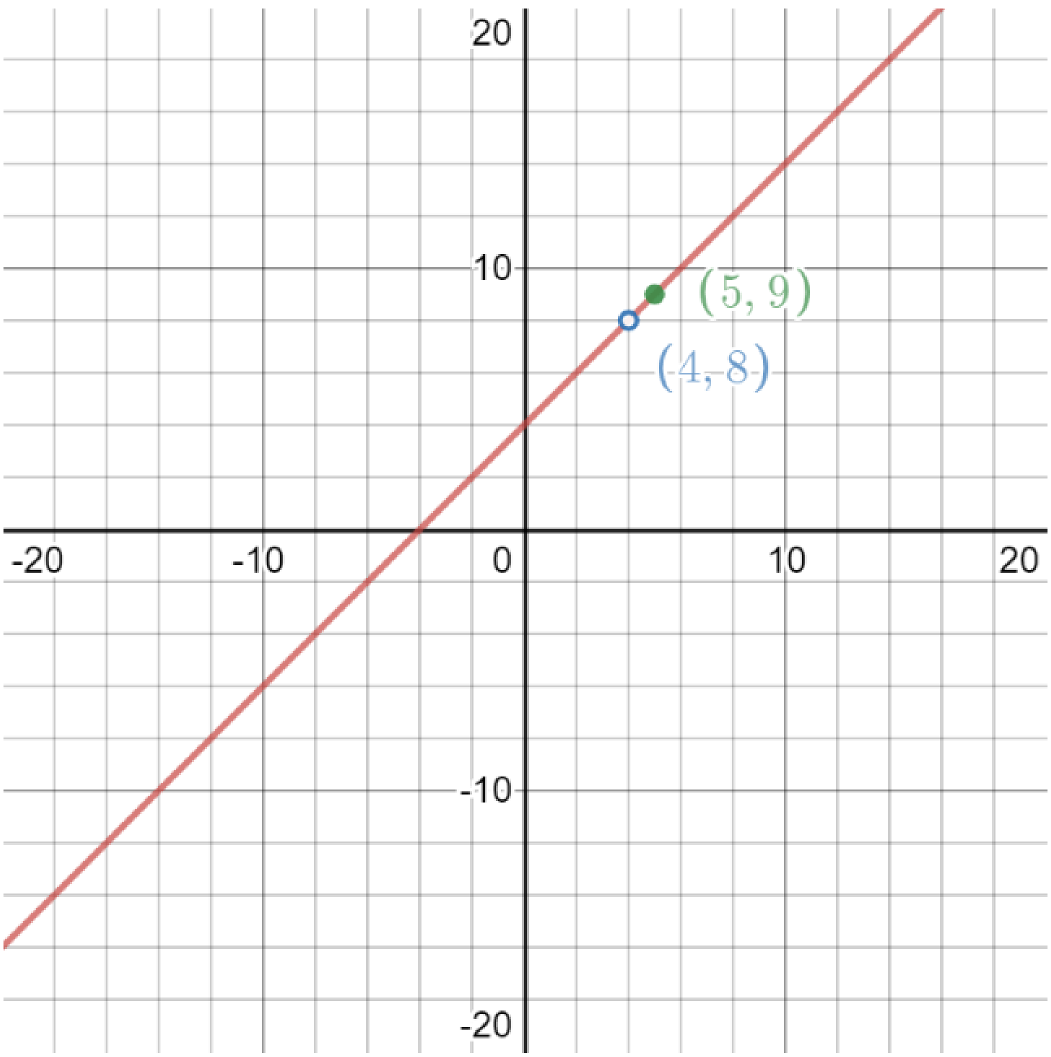
▪ The graphical process we just went through is called finding a **left-hand limit**.
▪ A left-hand limit is the process of finding the y-value that a function gets closer to starting with values of x that are smaller than the given value of x.

Let us repeat the process but start with values of x that are larger than x = 4, and then get progressively closer to x = 4. (this process is called finding a **right-hand limit**)

First: let x = 5 (5 is an arbitrary number I picked that is greater than 4)

$f(5) = \frac{5^2-16}{5-4} = 9$ (this gives the point (5,9)

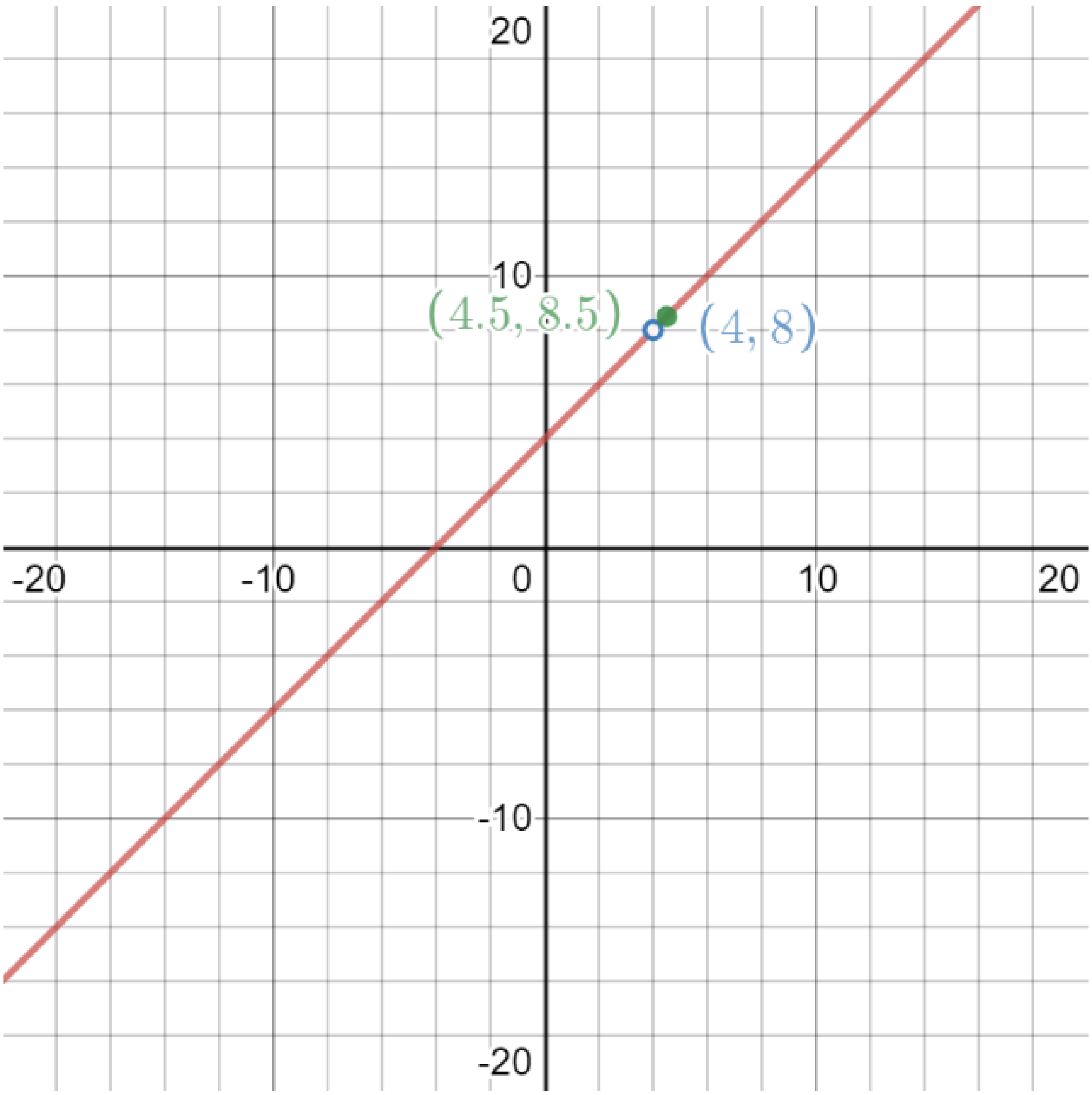
(you should notice that the point is not too far from the hole in graph)



Next: let x = 4.5 (4.5 is an arbitrary number I picked that is a bit closer to 4 then x = 5 was)

$f(4.5) = \frac{4.5^2-16}{4.5-4} = 8.5$ (graphically this gives the point (4.5, 8.5)

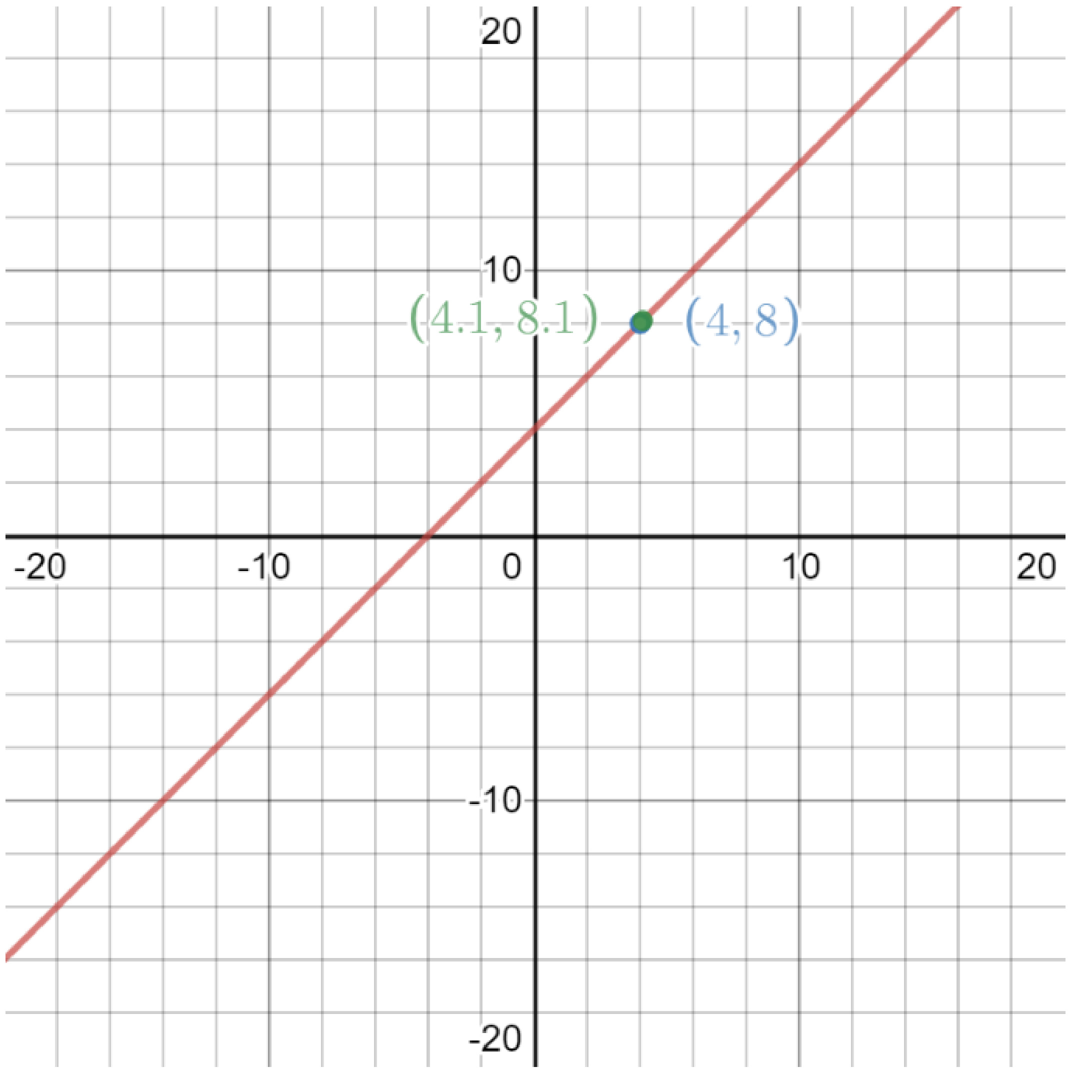
(you should notice that the point is even closer to the hole in the graph)



Next: let x = 4.1 (4.1 is an arbitrary number I picked that is a bit closer to 4 then x = 4.5 was)

$f(4.1) = \frac{4.1^2 - 16}{4.1 - 4} = 8.1$ (graphically this gives the point (4.1, 8.1))

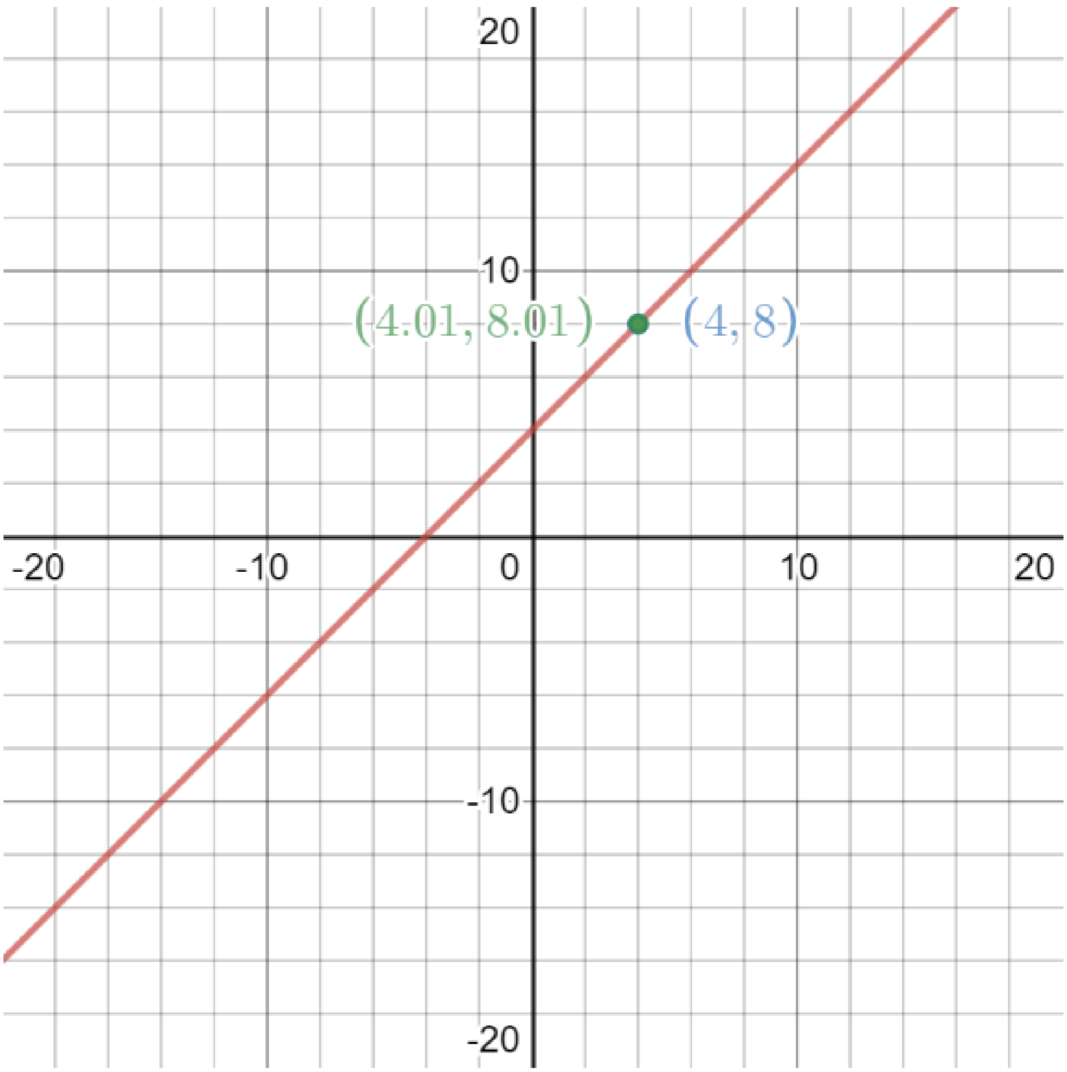
(you should notice that the point is even closer to the hole in the graph. In fact the point (4.1, 8.1) is so close to the hole that you cannot distinguish the two points.)



Next: let x = 4.01 (4.01 is an arbitrary number I picked that is a bit closer to 4 then x = 4.1 was)

$f(4.01) = \frac{4.01^2-16}{4.01-4} = 8.01$ (graphically this gives the point (4.01, 8.01)

(you should notice that the point is even closer to the hole in the graph. In fact the point (4.01, 8.01) is so close to the hole that you cannot distinguish the two points.)



Now I can make a conclusion about the y-value that my points are approaching as the x-values get closer to 4 (but remain larger than 4).

Each of these statements are equivalent:

- As the values of x that are **larger** than x = 4 get closer to x = 4 the y-values get closer to 8.
- $\lim_{x \to 4^+} \frac{x^2-16}{x-4} = 8$
- $\lim_{x \to 4^+} f(x) = 8$

- The graphical process we just went through is called finding a **right-hand limit**.
- A right-hand limit is the process of finding the y-value that a function gets closer to starting with values of x that are larger than the given value of x.

When: $\lim_{x \to 4^+} f(x) = \lim_{x \to 4^-} f(x) = 8$

That is when the left-hand limit and the right-hand limit equal the same number:

We say $\lim_{x \to 4} f(x) = 8$ (we remove the sign that indicates left / right-hand limit)

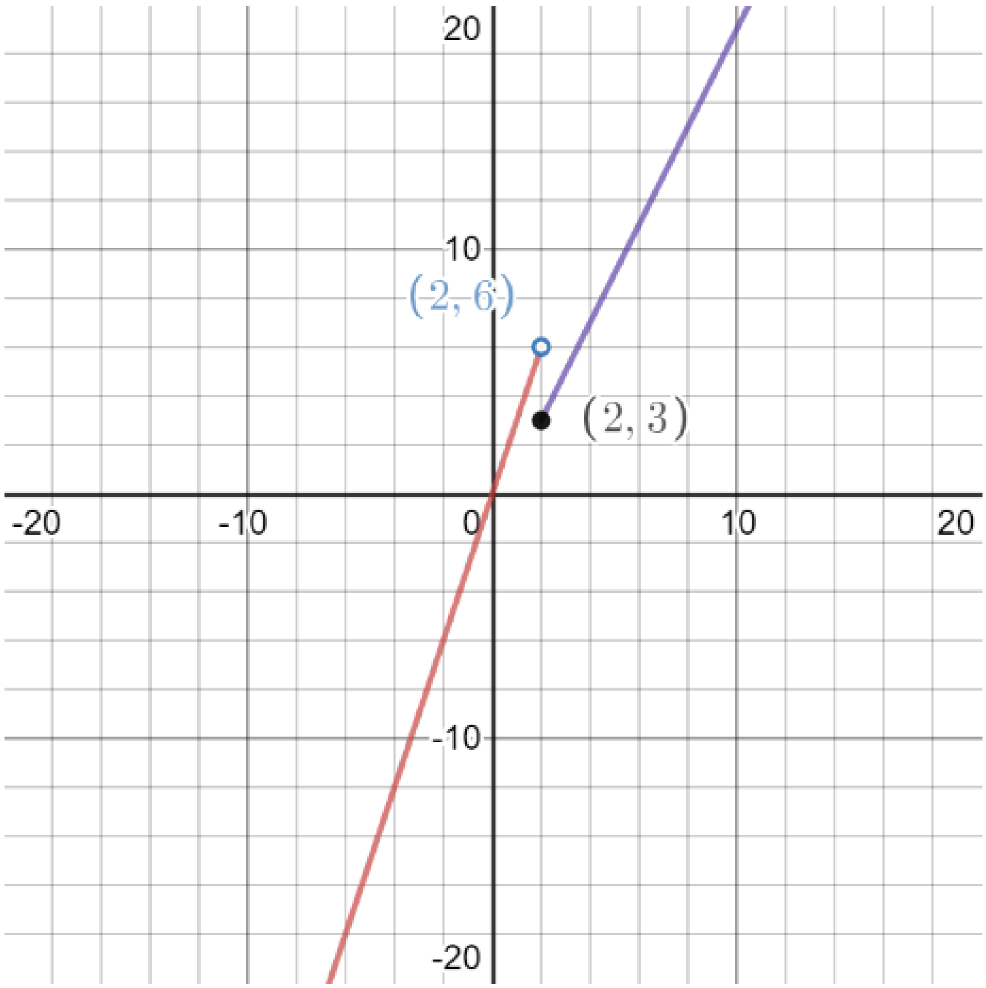
Both left-hand limits and right-hand limits are referred to as one-sided limits.

This symbol $\lim_{x \to 4} f(x)$ is called a two-sided limit.

Let us do another example of a piecewise defined function:

$$f(x) = \begin{cases} 3x, & if\ x < 2 \\ 2x - 1, & if\ x \geq 2 \end{cases}$$

Here is a graph of $f(x)$



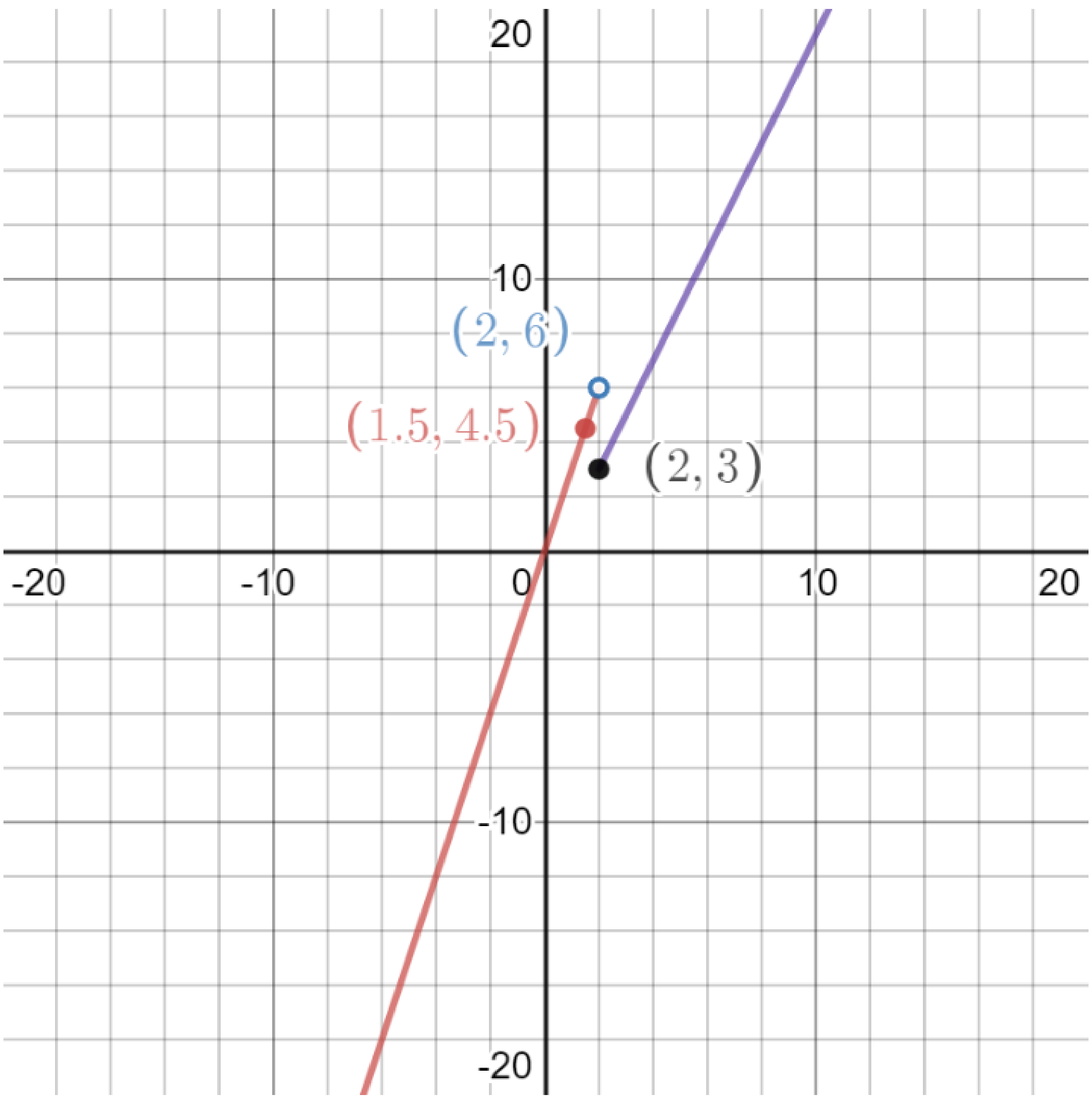
$$f(x) = \begin{cases} 3x, \ if \ x < 2 \\ 2x - 1, \ if \ x \geq 2 \end{cases}$$

Let us first find $\lim_{x \to 2^-} f(x)$

First: let x = 1.5 (1.5 is an arbitrary number I picked that is less than 2)

$f(1.5) = 3(1.5) =$
$4.5 \ plug \ in \ top \ function \ since \ 1.5 \ is \ less \ than \ 2$(this gives the point (1.5,4.5))

(you should notice that the point is not too far from the hole in graph)

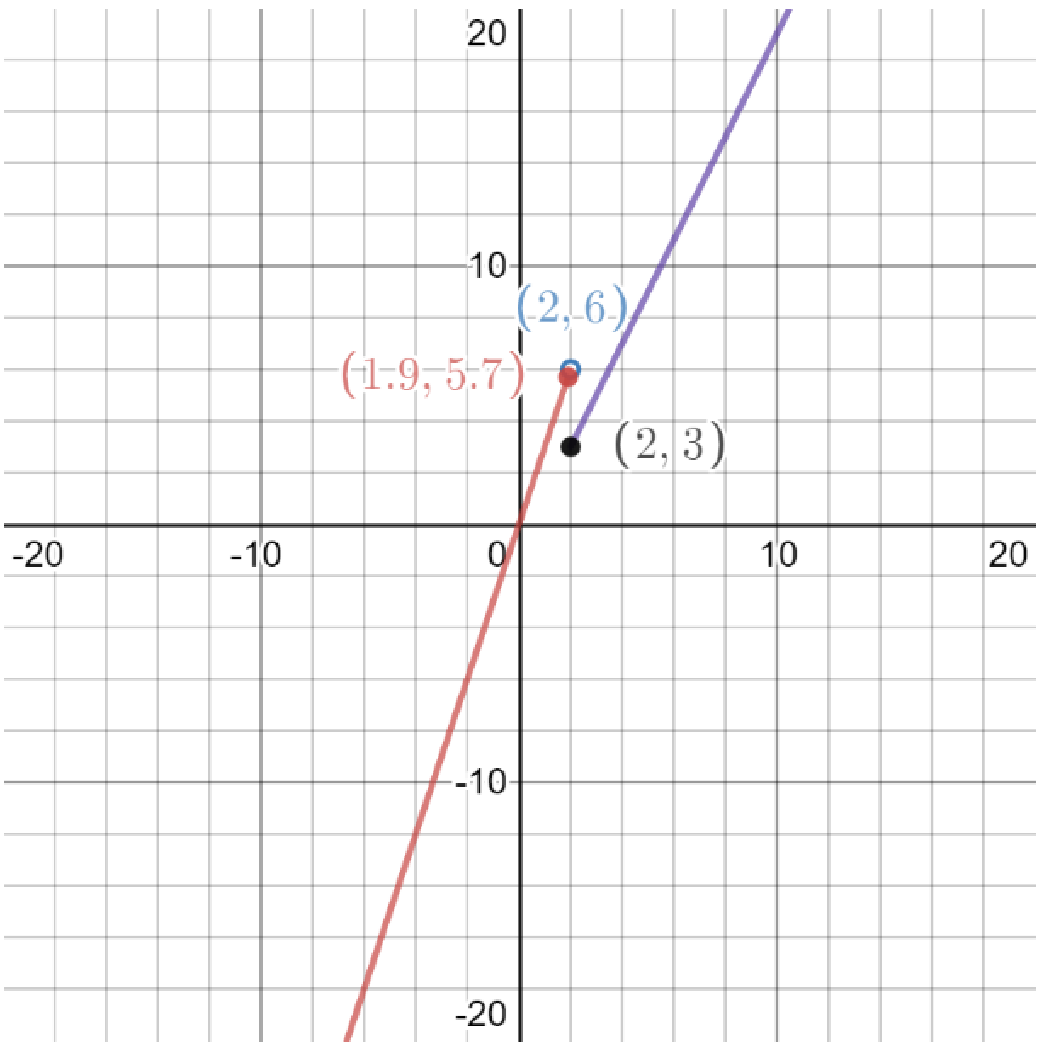


$$f(x) = \begin{cases} 3x, \ if \ x < 2 \\ 2x - 1, \ if \ x \geq 2 \end{cases}$$

Next: let x = 1.9 (1.9 is an arbitrary number I picked that is less than 2, but closer to 2 than x = 1.5)

$f(1.9) = 3(1.9) =$
$5.7 \ plug \ in \ top \ function \ since \ 1.9 \ is \ less \ than \ 2$(this gives the point (1.9,5.7))

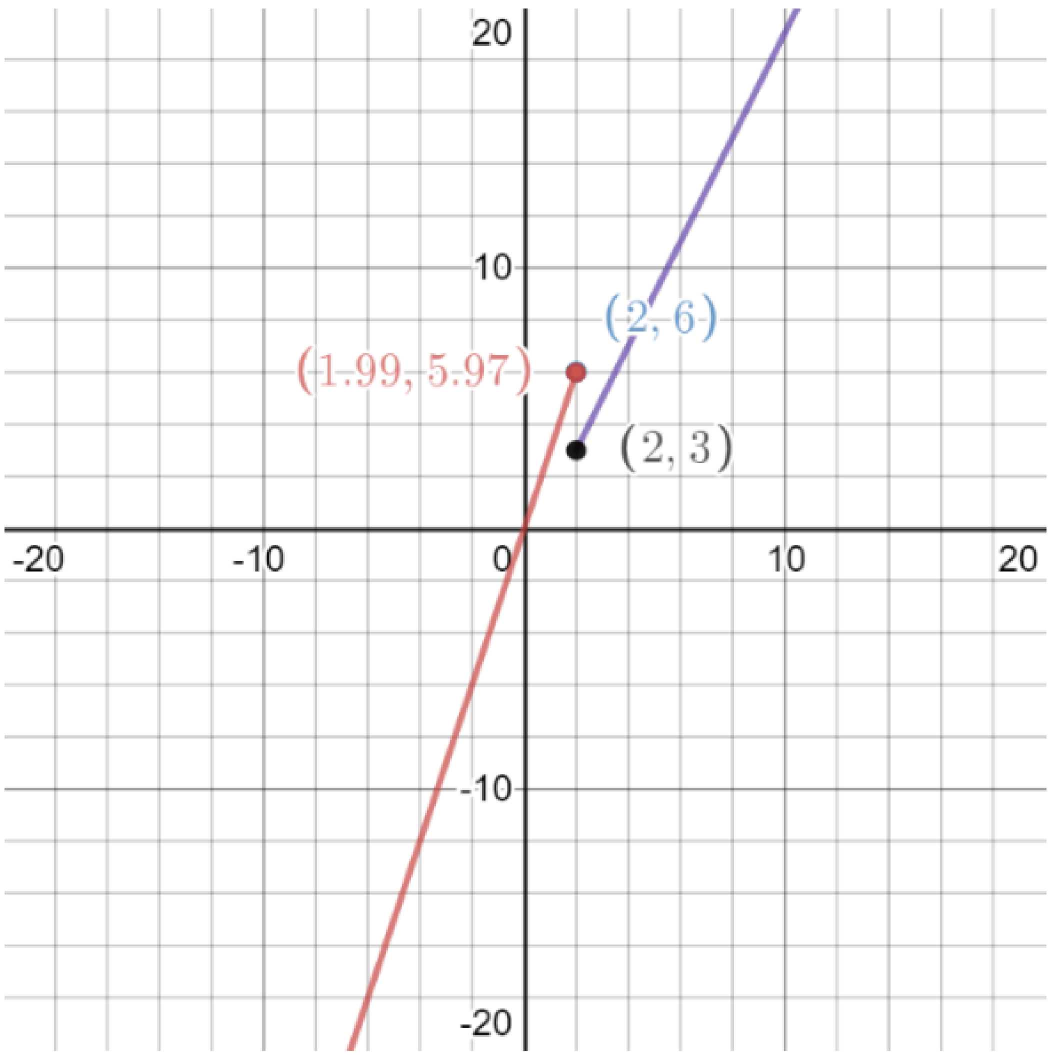
(you should notice that the point is even closer to the hole in graph)



$$f(x) = \begin{cases} 3x, & if\ x < 2 \\ 2x - 1, & if\ x \geq 2 \end{cases}$$

Next: let x = 1.99 (1.99 is an arbitrary number I picked that is less than 2, but closer to 2 than x = 1.9)

$f(1.99) = 3(1.99) =$
$5.97\ \ plug\ in\ top\ function\ since\ 1.99\ is\ less\ than\ 2$(this gives the point (1.99,5.97)) (you should notice that the point is even closer to the hole in graph)



Now I can make a conclusion about the y-value that my points are approaching as the x-values get closer to 2 (but remain smaller than 2).

Each of these statements are equivalent:

- As the values of x that are smaller than x = 2 get closer to x = 2 the y-values get closer to 6.

- $\lim_{x \to 2^-} f(x) = 6$

Important comments below:

- The graphical process we just went through is called finding a left-hand limit.
- A left-hand limit is the process of finding the y-value that a function gets closer to starting with values of x that are smaller than the given value of x.

$$f(x) = \begin{cases} 3x, \ if \ x < 2 \\ 2x - 1, \ if \ x \geq 2 \end{cases}$$

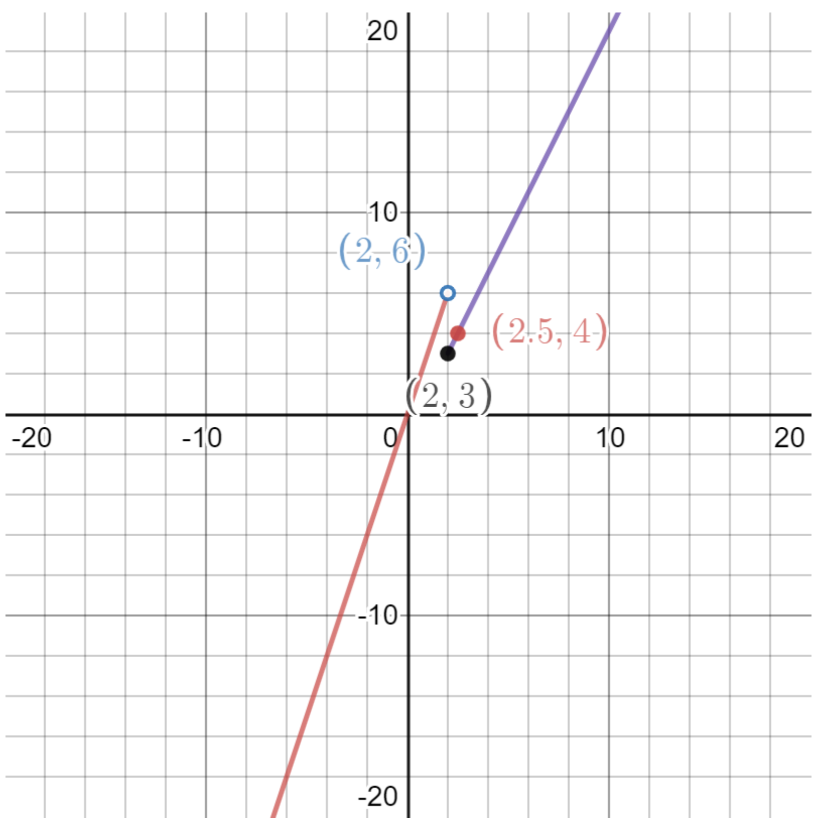
Now us first find $\lim_{x \to 2^+} f(x)$

First: let x = 2.5 (2.5 is an arbitrary number I picked that is greater than 2)

$f(2.5) = 2(2.5) - 1 = 4$

$plug\ in\ bottom\ function\ since\ 2.5\ is\ greater\ than\ 2$(this gives the point (2.5,4)

You should notice that the point is not too far from the point (2,3)



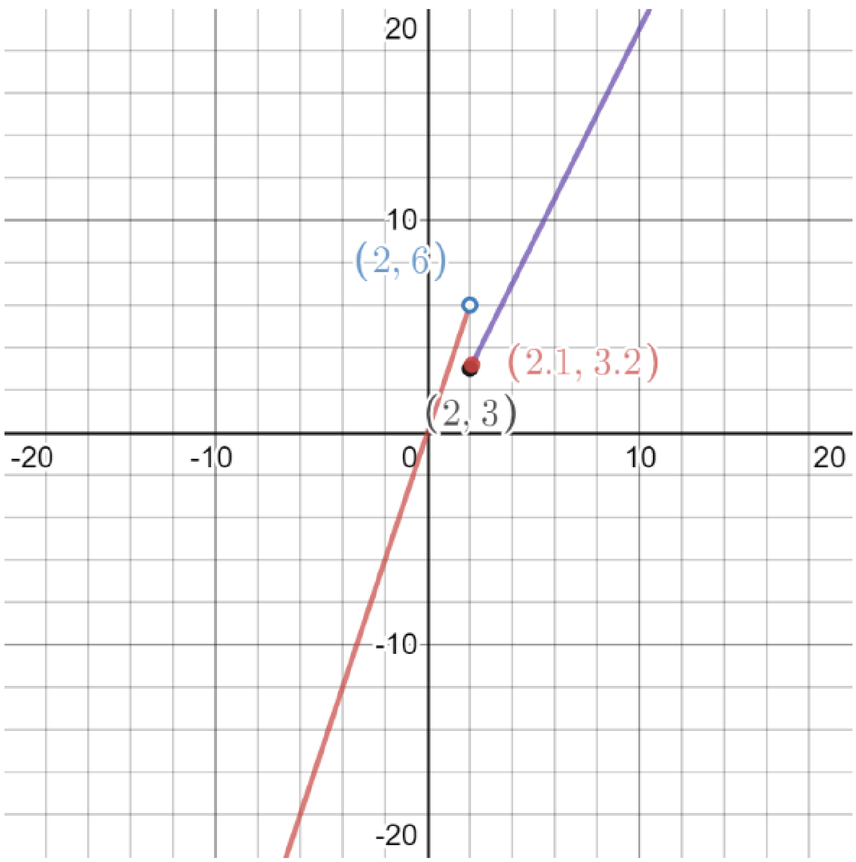
$$f(x) = \begin{cases} 3x, \ if \ x < 2 \\ 2x - 1, \ if \ x \geq 2 \end{cases}$$

Next: let x = 2.1 (2.1 is an arbitrary number I picked that is greater than 2, but closer to 2 than 2.5 is)

$f(2.1) = 2(2.1) - 1 =$
$3.2 \ plug \ in \ bottom \ function \ since \ 2.1 \ is \ greater \ than \ 2$

this gives the point (2.1,3.2)

(you should notice that the point is even closer to the point (2,3)



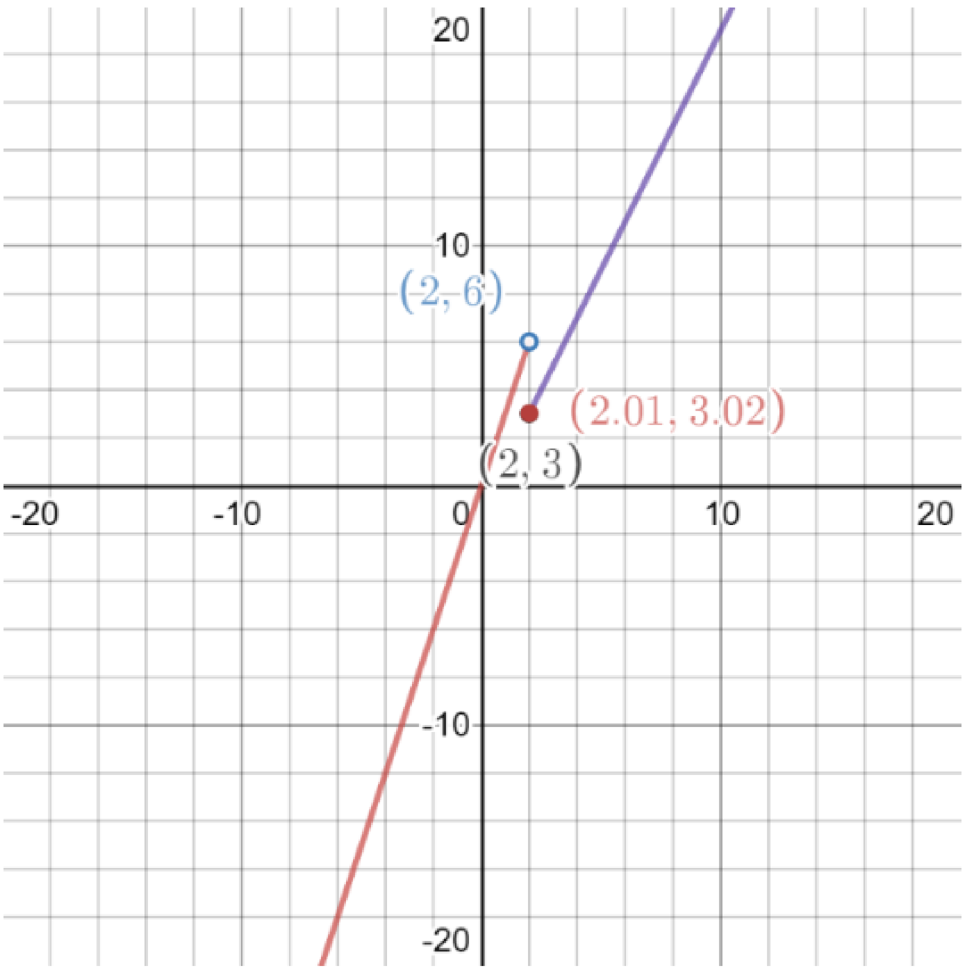
$$f(x) = \begin{cases} 3x, \ if \ x < 2 \\ 2x - 1, \ if \ x \geq 2 \end{cases}$$

Next: let x = 2.01 (2.01 is an arbitrary number I picked that is greater than 2, but closer to 2 than 2.1 is)

$f(2.01) = 2(2.01) - 1 =$
$3.02 \ plug \ in \ bottom \ function \ since \ 2.01 \ is \ greater \ than \ 2$

this gives the point (2.01,3.02)

(you should notice that the point is even closer to the point (2,3)



Now I can make a conclusion about the y-value that my points are approaching as the x-values get closer to 2 (but remain larger than 2).

Each of these statements are equivalent:

- As the values of x that are larger than x = 2 get closer to x = 2 the y-values get closer to 3.
- $\lim_{x \to 2^+} f(x) = 3$

Important comments

- The graphical process we just went through is called finding a right-hand limit.
- A right-hand limit is the process of finding the y-value that a function gets closer to starting with values of x that are larger than the given value of x.

In this example (both of these are called one-side limits)

- $\lim_{x \to 2^-} f(x) = 6$
- $\lim_{x \to 2^+} f(x) = 3$

Unlike the first example the left-hand limit and right-hand limit are different numbers.
When this happens. we say

$\lim_{x \to 2} f(x) = does\ not\ exist\ \ (dne)$ (this is called a 2-sided limit)

A TWO-SIDED LIMIT ONLY EXISTS WHEN THE LEFT AND RIGHT-HAND LIMITS ARE EQUAL!!

Here is a semi-formal definition of the concept of a two-sided limit:

$$\lim_{x \to a} f(x) = L$$

The two sided limit of the function $f(x)$ as x approaches some value x = a is equal to y = L, provided the y-values get arbitrarily close to L as the x-values get sufficiently close to x = a.

Limits at Infinity and Horizontal Asymptotes

We can extend the idea of a limit at a value of x = a to limits at x = infinity.

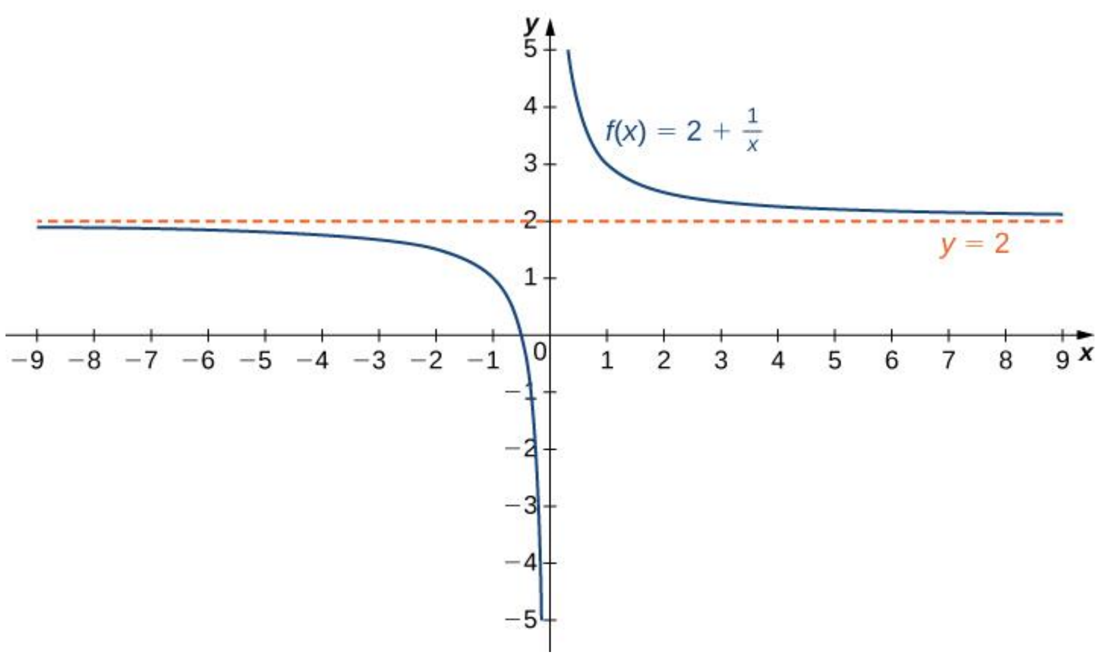
For example, consider the graph of the function $f(x) = 2 + \frac{1}{x}$

We can see as the values of x get larger and approach "∞" the y-values of the function $f(x)$ approach y = 2.

We say: $\lim_{x \to \infty} f(x) = 2$ (this is a one-sided limit, as there are no numbers greater than ∞)

Similarly, we can see as the values of x get smaller and approach "$-\infty$" the y-values of the function $f(x)$ also approach y = 2.

We say: $\lim_{x \to -\infty} f(x) = 2$ (this is a one-sided limit as there are no numbers less than $-\infty$)



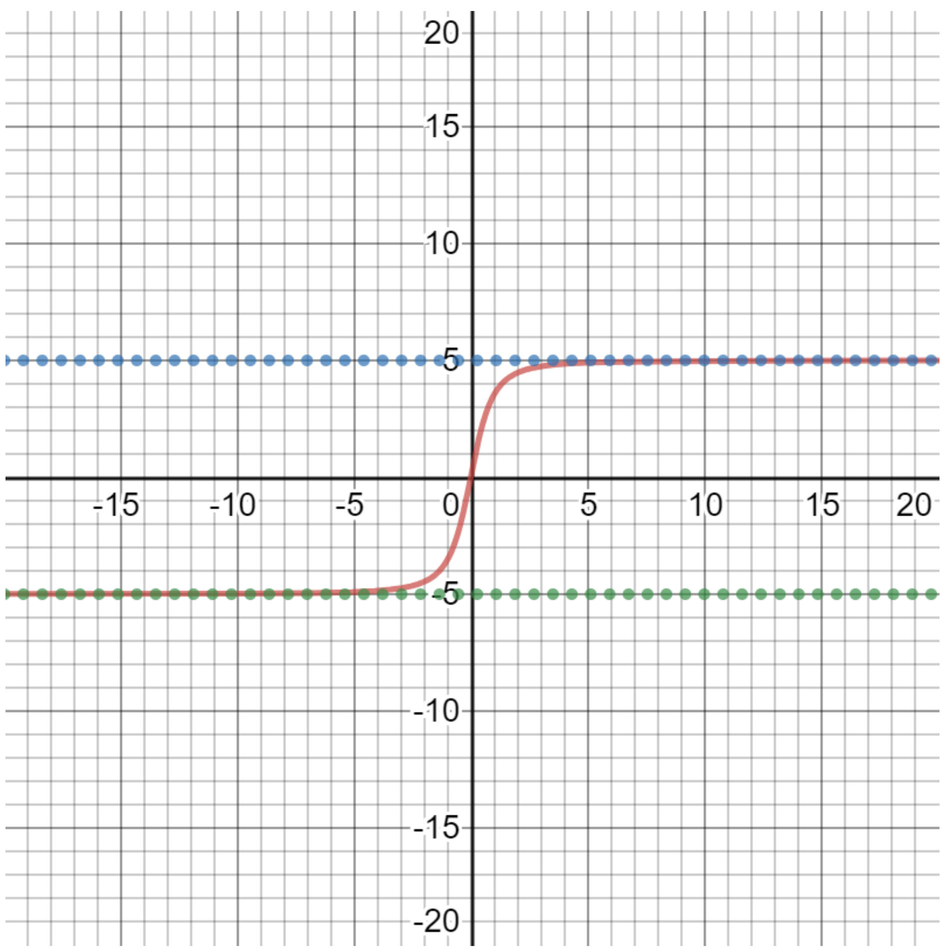
For example, consider the function $f(x) = \frac{5x}{\sqrt{x^2+1}}$

We can see as the values of x get larger and approach "∞" the y-values of the function $f(x)$ approach y = 5.

We say: $\lim_{x \to \infty} f(x) = 5$

Similarly, we can see as the values of x get smaller and approach "$-\infty$" the y-values of the function $f(x)$ approach y = -5.

We say: $\lim_{x \to -\infty} f(x) = -5$



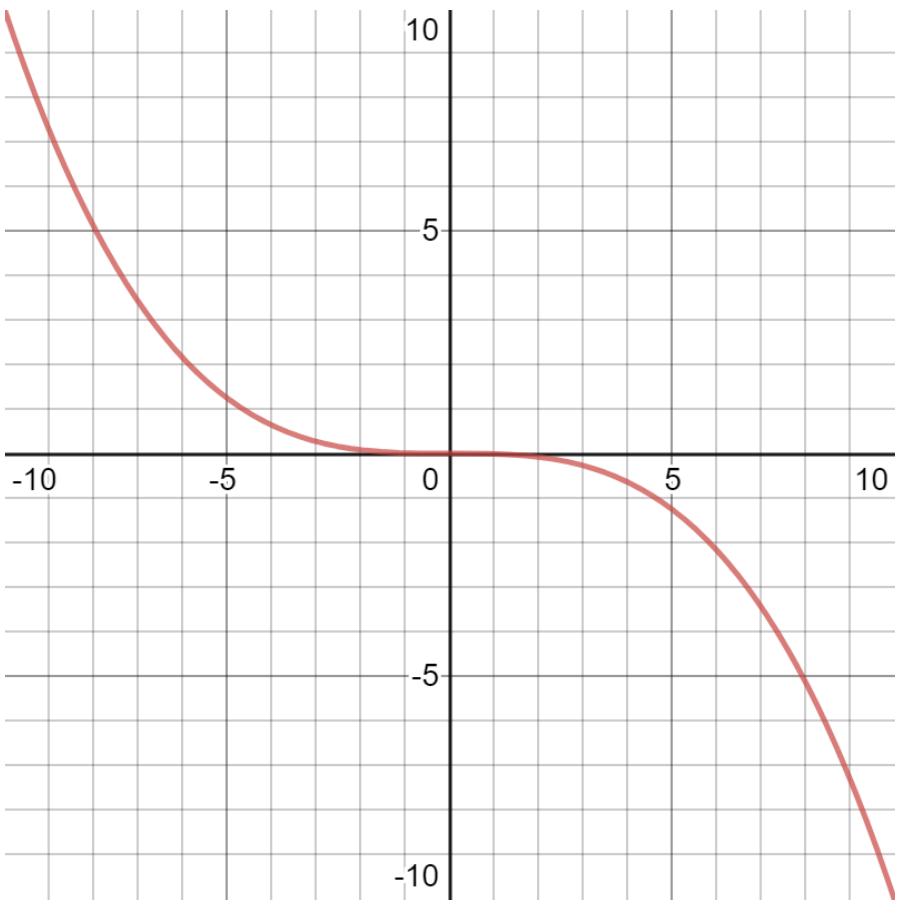
For example, consider the function $f(x) = -.01x^3$

We can see as the values of x get larger and approach "∞" the y-values of the function $f(x)$ don't approach any horizontal line. In fact, the y-values get continually smaller.

We say: $\lim_{x\to\infty} f(x) = -\infty$

We can see as the values of x get smaller and approach "$-\infty$" the y-values of the function $f(x)$ don't approach any horizontal line. In fact, the y-values get continually larger.

We say: $\lim_{x\to-\infty} f(x) = \infty$



So far, we have focused on graphs to find limits. We can also use tables to find limits:

For example: (I will complete the table and write in comments when I record a video)

Complete the table and estimate $f(x) = \lim_{x \to 2^-} (3x + 1)$

(notice the x-values in the table are smaller than 2 but get closer and closer to x = 2)

x	1.5	1.9	1.99	1.999
$f(x)$				

Complete the table and estimate $f(x) = \lim_{x \to 2^+} (3x + 1)$

(notice the x-values in the table are larger than 2 but get closer and closer to x = 2)

x	2.5	2.1	2.01	2.001
$f(x)$				

Use the results to estimate: $f(x) = \lim_{x \to 2} (3x + 1)$

Here is another example of using a table to find limits:

For example: (I will complete the table and write in comments when I record a video)

Complete the table and estimate $f(x) = \lim_{x \to -3^-} \frac{|x+3|}{x+3} + x$

(notice the x-values in the table are smaller than 2 but get closer and closer to x = 2)

x	-3.5	-3.1	-3.01	-3.001
$f(x)$				

Complete the table and estimate $f(x) = \lim_{x \to -3^+} \frac{|x+3|}{x+3} + x$

(notice the x-values in the table are larger than 2 but get closer and closer to x = 2)

x	-2.5	-2.9	-2.99	-2.999
$f(x)$				

Use the results to estimate: $f(x) = \lim_{x \to 3} \frac{|x+3|}{x+3} + x$

Here is another example of using a table to find limits (this time at $x = \infty$):

For example: (I will complete the table and write in comments when I record a video)

Complete the table and estimate $f(x) = \lim_{x \to \infty} \frac{8x+6}{2x-1}$

(notice the x-values in the table are smaller than ∞ but get closer and closer to x = ∞)

(this must be a one-sided limit as there no numbers I can use that are larger than infinity)

x	100	1000	10,000	100,000
$f(x)$				

Here is another example of using a table to find limits (this time at $x = \infty$):

For example: (I will complete the table and write in comments when I record a video)

Complete the table and estimate $f(x) = \lim_{x \to \infty} (-2x + 24)$

(notice the x-values in the table are smaller than ∞ but get closer and closer to x = ∞)

(this must be a one-sided limit as there no numbers I can use that are larger than infinity)

x	100	1000	10,000	100,000
$f(x)$				

Here is another example of using a table to find limits (this time at $x = \infty$):

For example: (I will complete the table and write in comments when I record a video)

Complete the table and estimate

$$f(x) = \lim_{x \to \infty} \left(\frac{-3x^2+1}{x-4} \right)$$

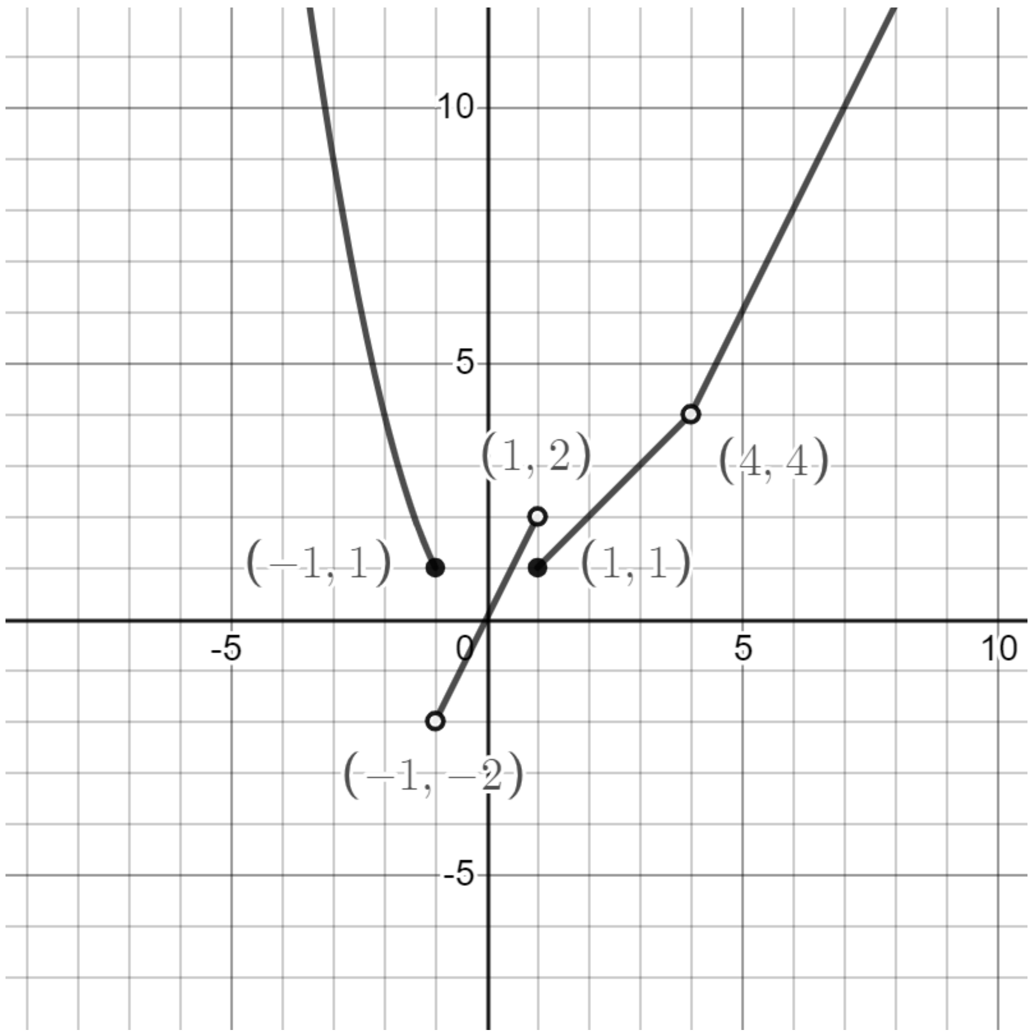
(notice the x-values in the table are smaller than ∞ but get closer and closer to x = ∞)

(this must be a one-sided limit as there no numbers I can use that are larger than infinity)

x	100	1000	10,000
$f(x)$			

(minimum homework: 1.1 1-11 odds, 15, 19 and 21)

1) Below is a graph of the function f(x).



Find the following

a) f(1)

b) f(-1)

c) f(4)

d) $\lim_{x \to 1^-} f(x)$

e) $\lim_{x \to 1^+} f(x)$

f) $\lim_{x \to 1} f(x)$

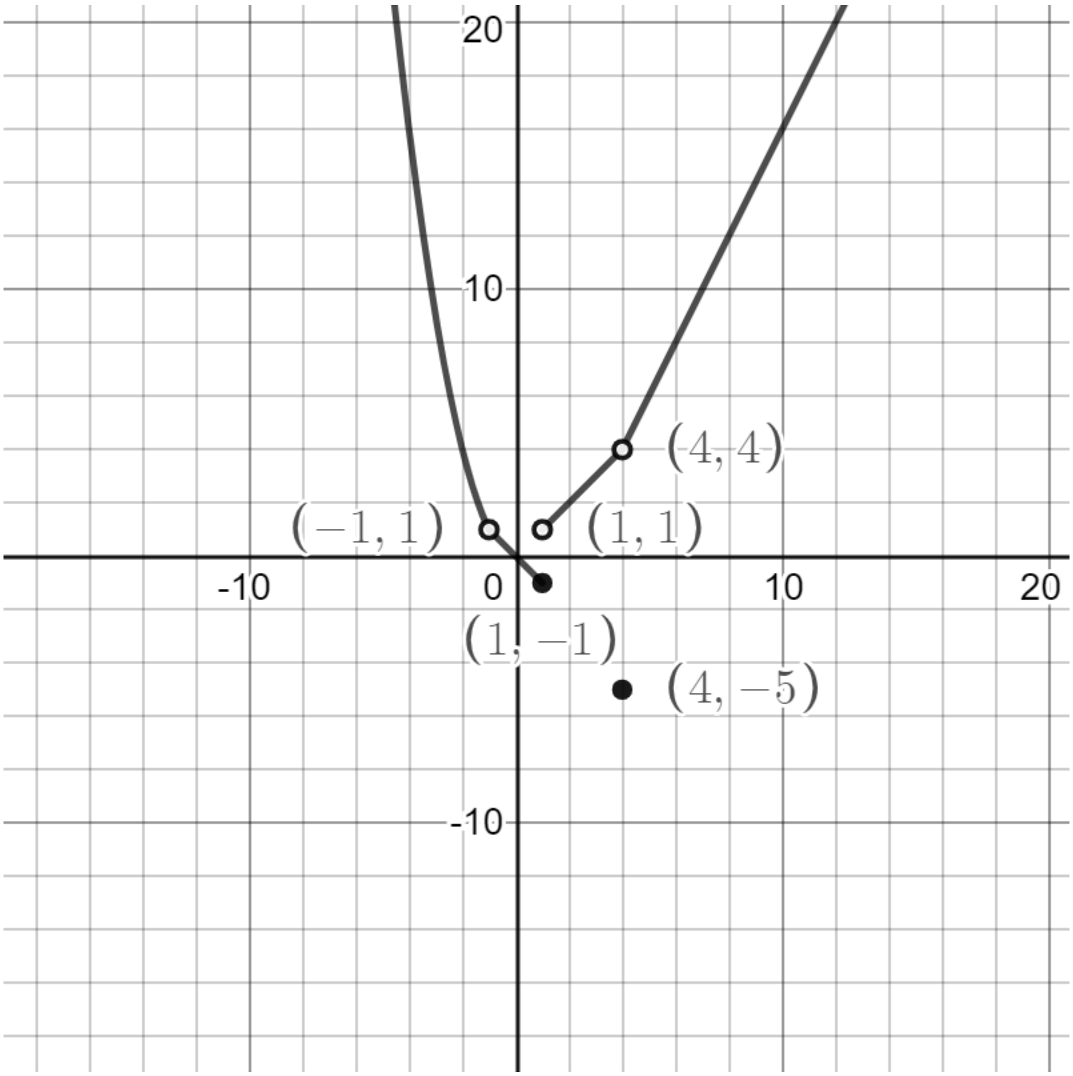
g) $\lim_{x \to 4^-} f(x)$

h) $\lim_{x \to 4^+} f(x)$

i) $\lim_{x \to 4} f(x)$

(minimum homework: 1.1 1-11 odds, 15, 19 and 21)

2) Below is a graph of the function f(x).



Find the following:

a) f(1)

b) f(-1)

c) f(4)

d) $\lim_{x\to 1^-} f(x)$

e) $\lim_{x\to 1^+} f(x)$

f) $\lim_{x\to 1} f(x)$

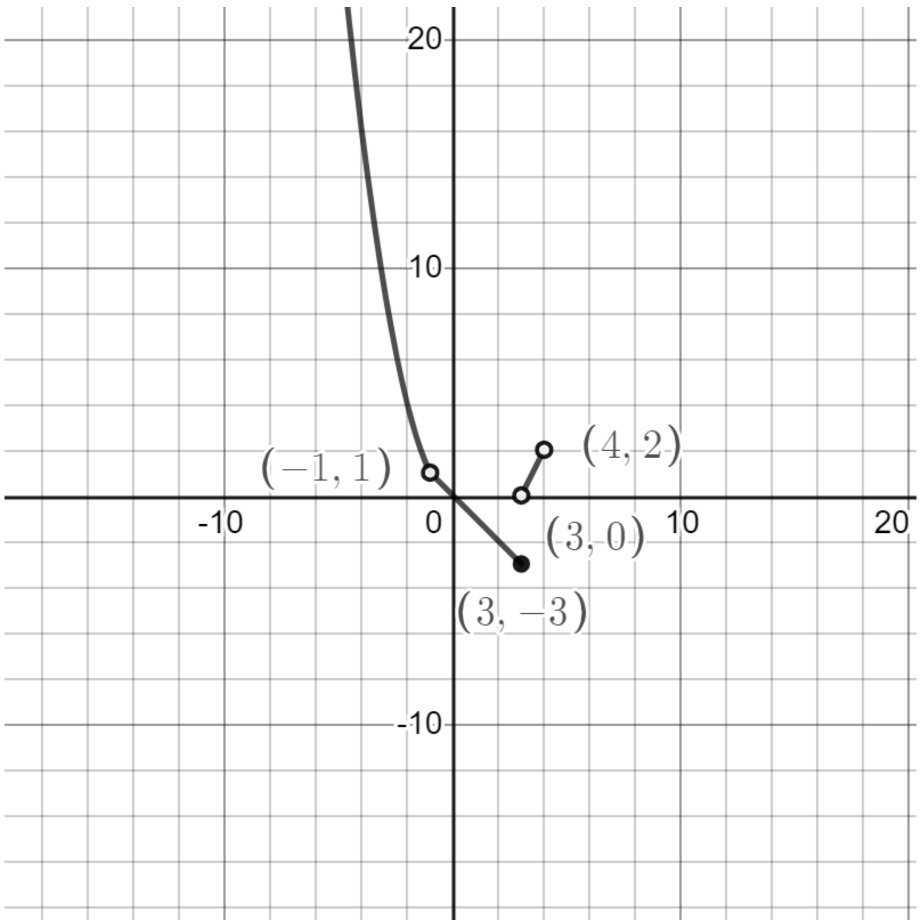
g) $\lim_{x\to 4-} f(x)$

h) $\lim_{x\to 4^+} f(x)$

i) $\lim_{x\to 4} f(x)$

(minimum homework: 1.1 1-11 odds, 15, 19 and 21)

3) Below is a graph of the function f(x).



Find the following

a) f(3)

b) f(4)

c) f(-1)

d) $\lim_{x \to -1^-} f(x)$

e) $\lim_{x \to -1^+} f(x)$

f) $\lim_{x \to -1} f(x)$

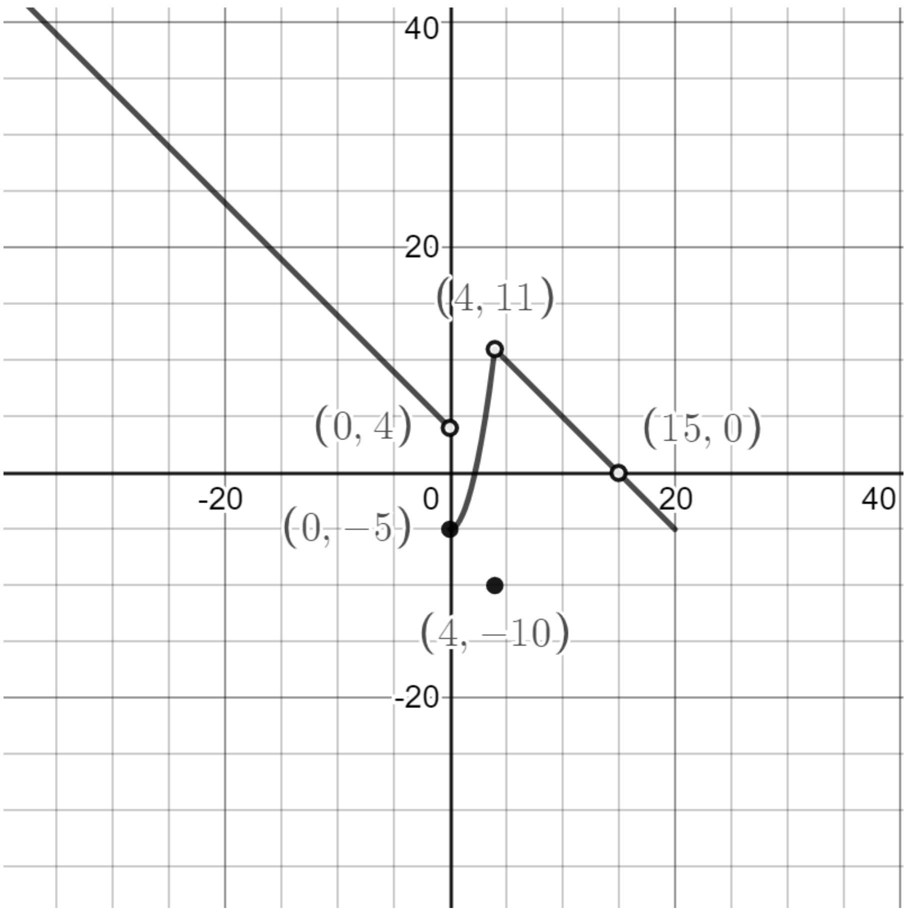
g) $\lim_{x \to 3^-} f(x)$

h) $\lim_{x \to 3^+} f(x)$

i) $\lim_{x \to 3} f(x)$

(minimum homework: 1.1 1-11 odds, 15, 19 and 21)

4) Below is the graph of a function y = f(x).



Find the following

a) f(0)

b) f(4)

c) f(15)

d) $\lim_{x \to 4^-} f(x)$

e) $\lim_{x \to 4^+} f(x)$

f) $\lim_{x \to 4} f(x)$

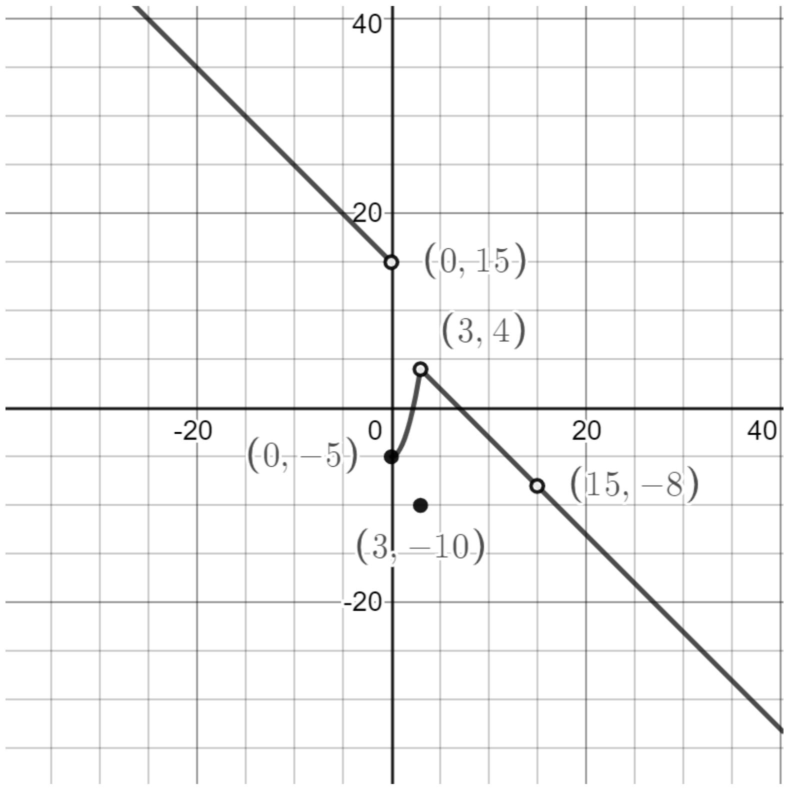
g) $\lim_{x \to 0^-} f(x)$

h) $\lim_{x \to 0^+} f(x)$

i) $\lim_{x \to 0} f(x)$

(minimum homework: 1.1 1-11 odds, 15, 19 and 21)

5) Below is a graph of the function f(x).



Find the following:

a) f(0)

b) f(3)

c) f(15)

d) $\lim_{x\to 3^-} f(x)$

e) $\lim_{x\to 3^+} f(x)$

f) $\lim_{x\to 3} f(x)$

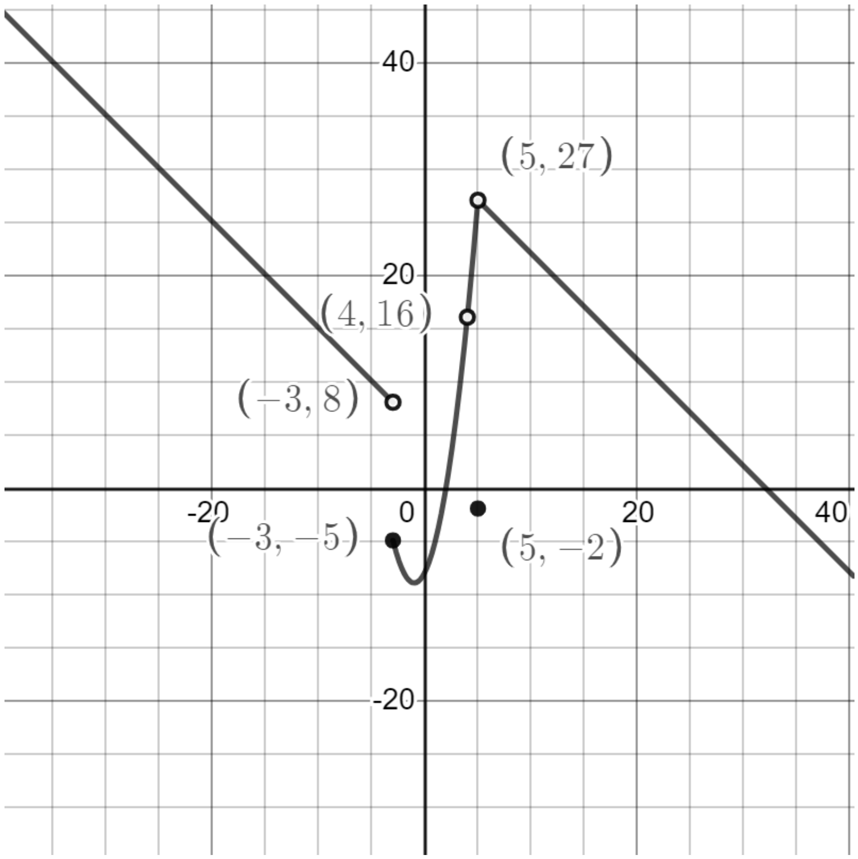
g) $\lim_{x\to 0^-} f(x)$

h) $\lim_{x\to 0^+} f(x)$

i) $\lim_{x\to 0} f(x)$

(minimum homework: 1.1 1-11 odds, 15, 19 and 21)

6) Below is a graph of the function f(x).



Find the following:

a) f(5)

b) f(-3)

c) f(4)

d) $\lim_{x \to -3^-} f(x)$

e) $\lim_{x \to -3^+} f(x)$

f) $\lim_{x \to -3} f(x)$

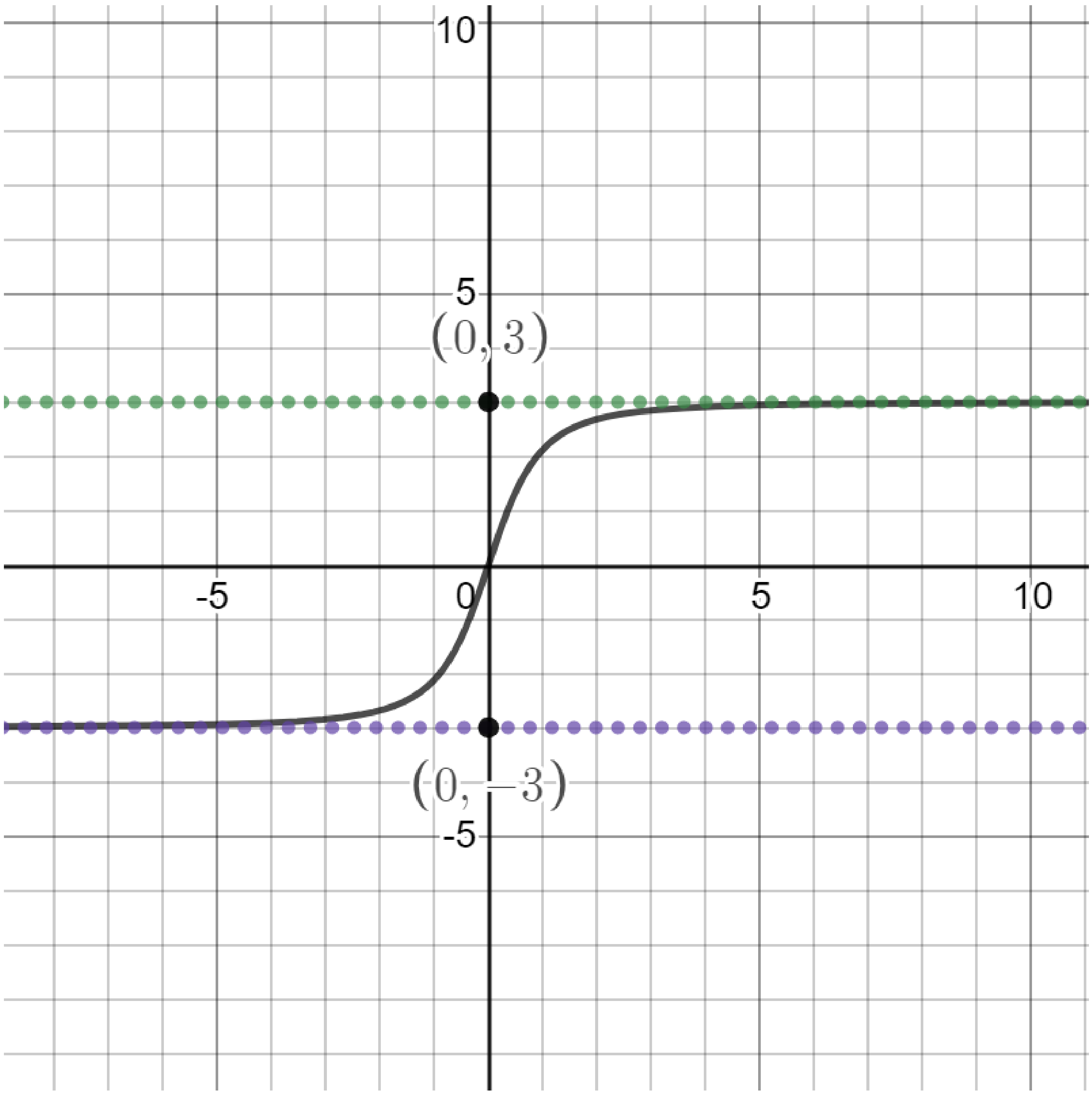
g) $\lim_{x \to 5^-} f(x)$

h) $\lim_{x \to 5^+} f(x)$

i) $\lim_{x \to 5} f(x)$

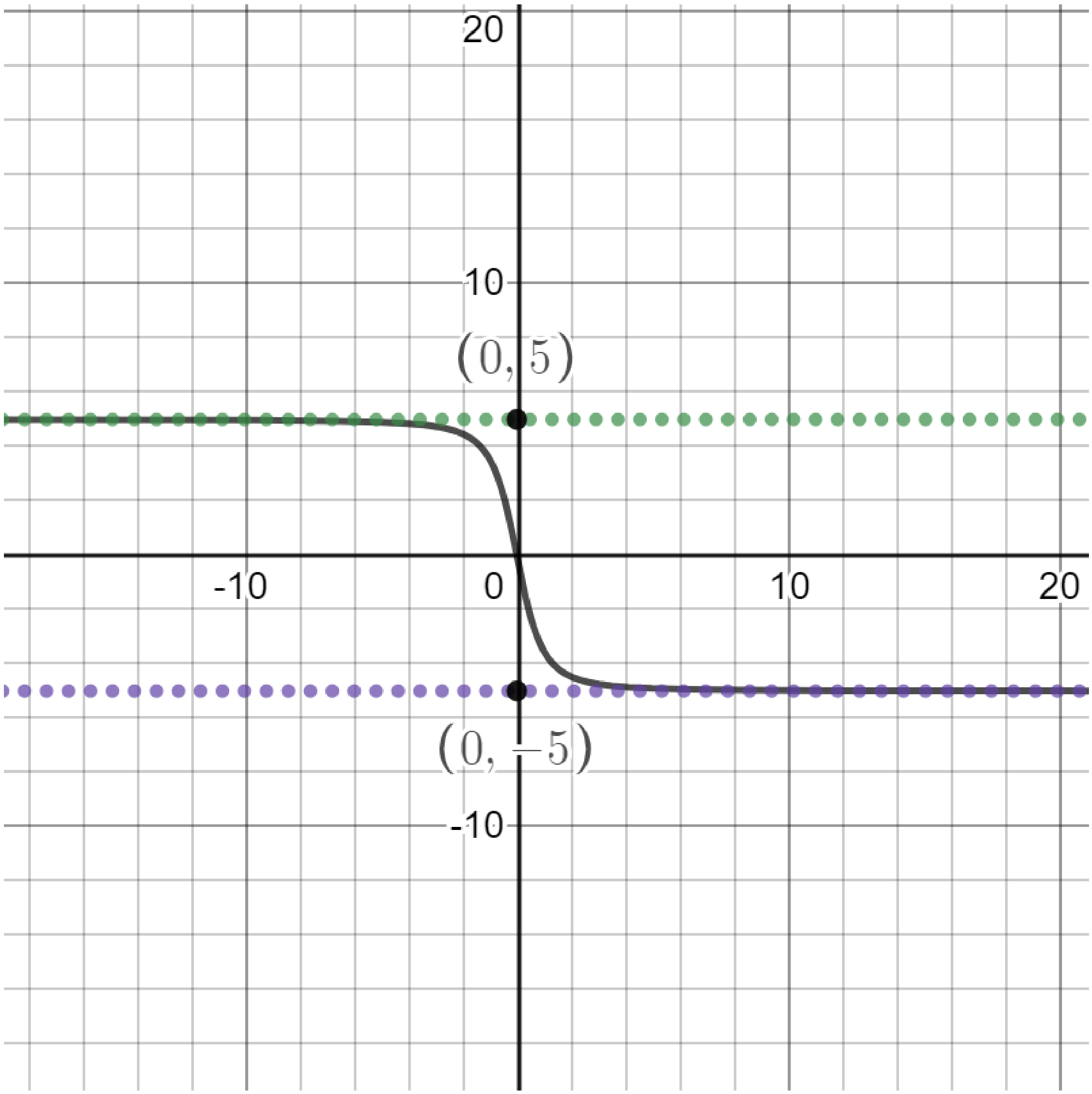
(minimum homework: 1.1 1-11 odds, 15, 19 and 21)

7) Below is a graph of the function f(x). Find the value of each limit (if it exists)



a) $\lim_{x\to\infty} f(x)$ b) $\lim_{x\to-\infty} f(x)$

8) Below is a graph of the function f(x). Find the value of each limit (if it exists)

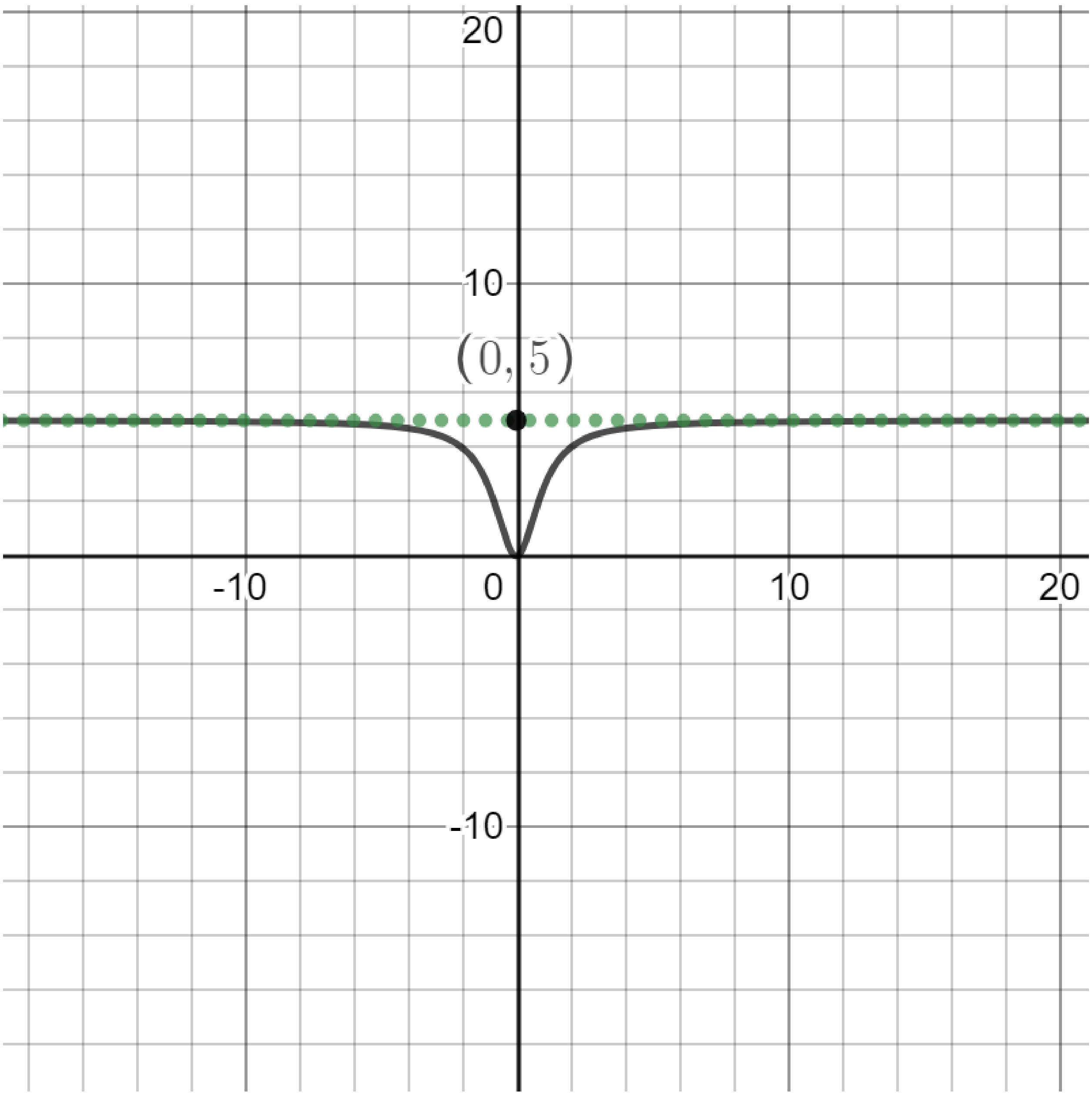


a) $\lim_{x \to \infty} f(x)$

b) $\lim_{x \to -\infty} f(x)$

(minimum homework: 1.1 1-11 odds, 15, 19 and 21)

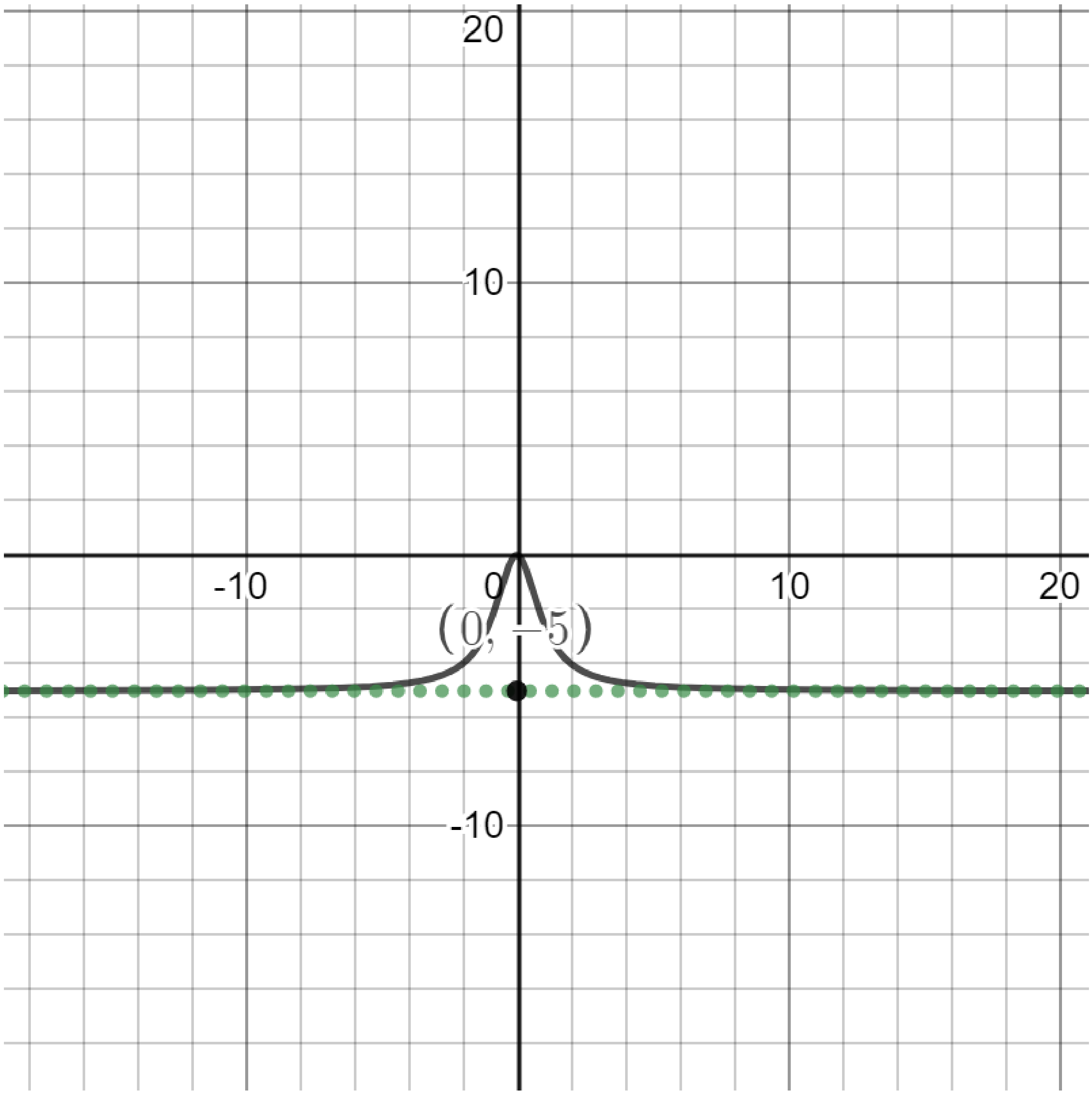
9) Below is a graph of the function f(x). Find the value of each limit (if it exists)



a) $\lim_{x\to\infty} f(x)$

b) $\lim_{x\to-\infty} f(x)$

10) Below is a graph of the function f(x). Find the value of each limit (if it exists)

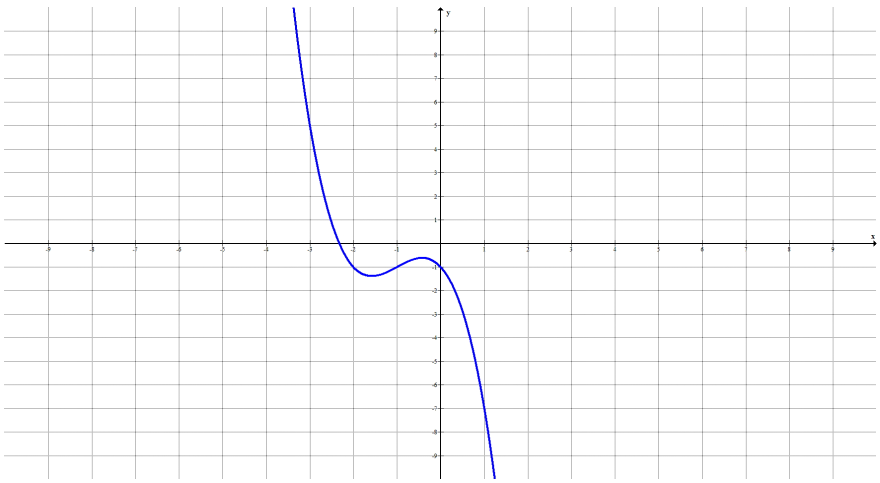


a) $\lim_{x \to \infty} f(x)$

b) $\lim_{x \to -\infty} f(x)$

(minimum homework: 1.1 1-11 odds, 15, 19 and 21)

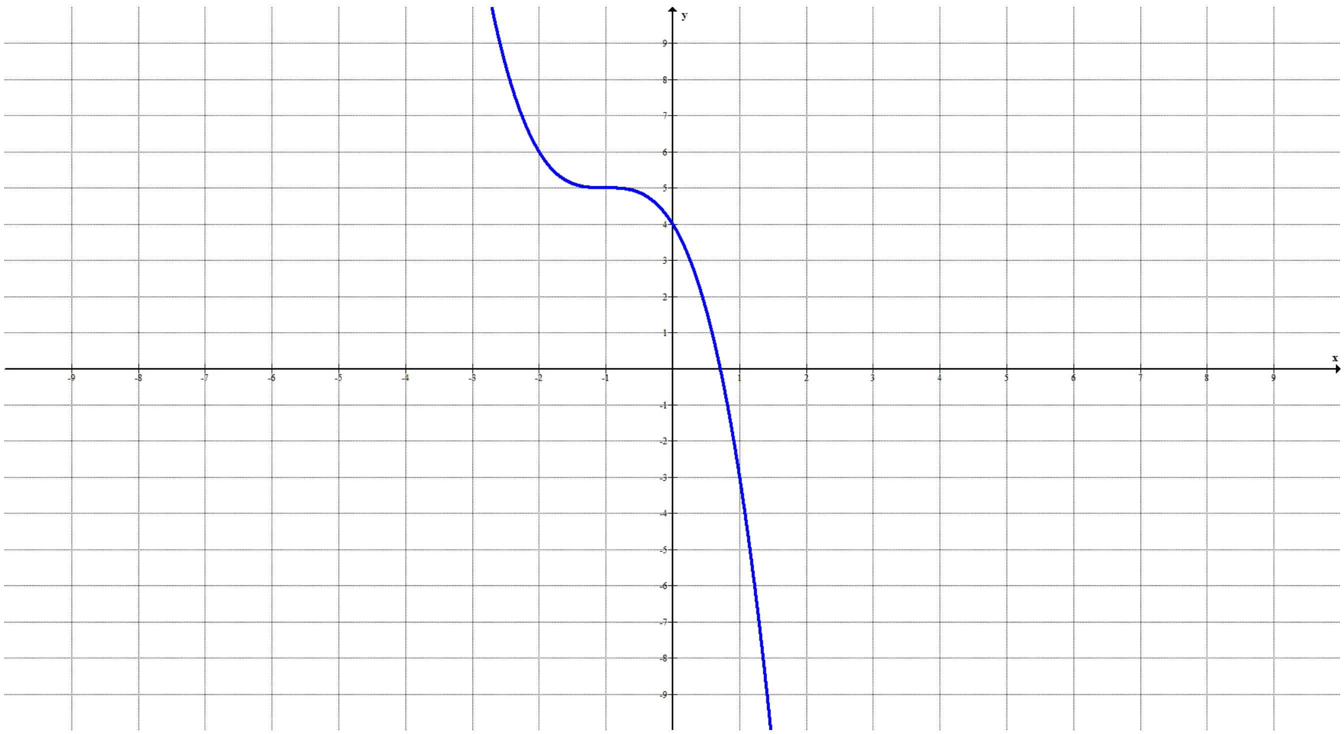
11) Below is a graph of the function f(x). Find the value of each limit (if it exists)



a) $\lim_{x \to \infty} f(x)$

b) $\lim_{x \to -\infty} f(x)$

12) Below is a graph of the function f(x). Find the value of each limit (if it exists)



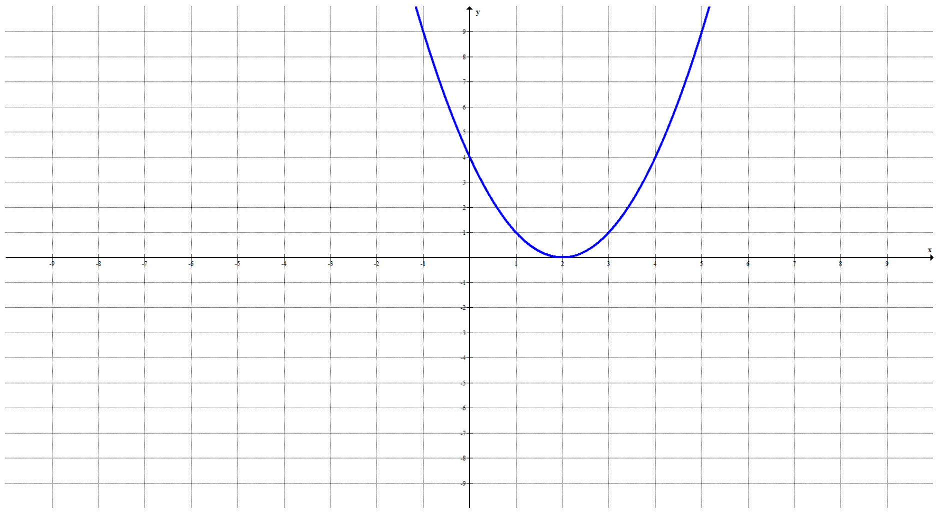
a) $\lim_{x \to \infty} f(x)$

b) $\lim_{x \to -\infty} f(x)$

Answers: a) $\lim_{x \to \infty} f(x) = -\infty$ b) $\lim_{x \to -\infty} f(x) = \infty$

(minimum homework: 1.1 1-11 odds, 15, 19 and 21)

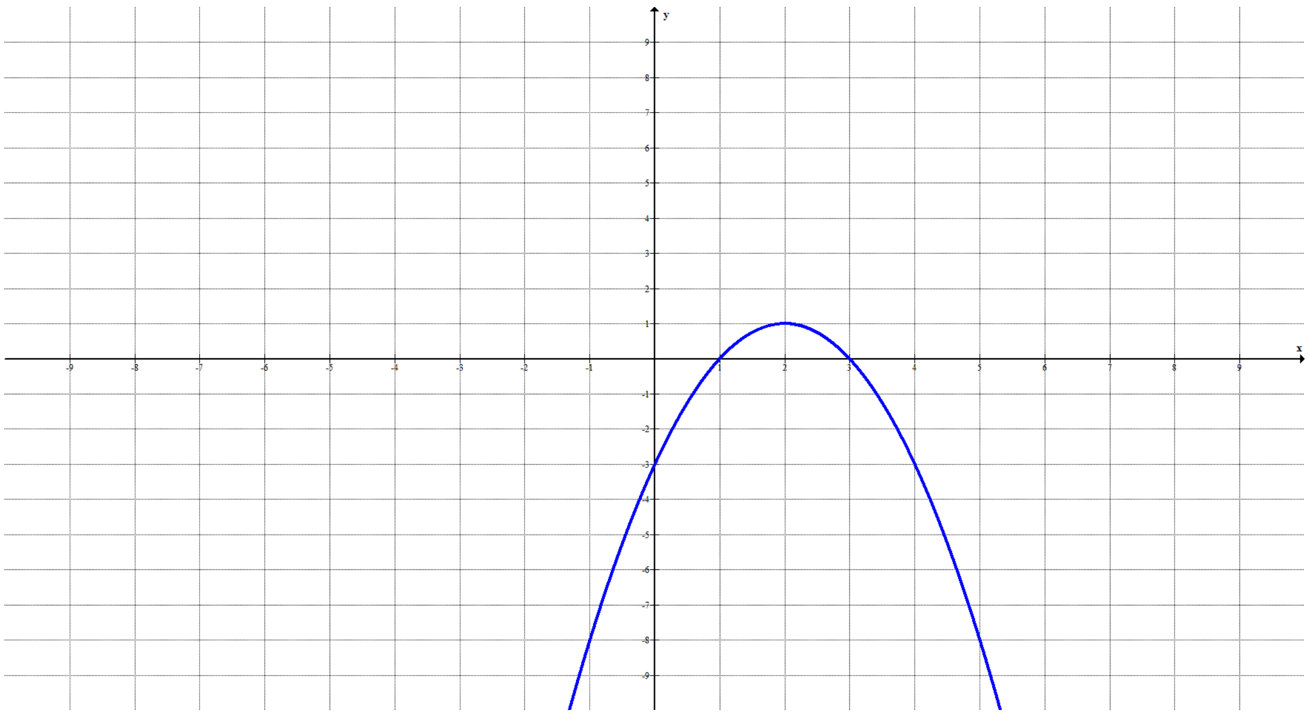
13) Below is a graph of the function f(x). Find the value of each limit (if it exists)



a) $\lim_{x \to \infty} f(x)$

b) $\lim_{x \to -\infty} f(x)$

14) Below is a graph of the function f(x). Find the value of each limit (if it exists)



a) $\lim_{x \to \infty} f(x)$

b) $\lim_{x \to -\infty} f(x)$

Answers: a) $\lim_{x \to \infty} f(x) = -\infty$ b) $\lim_{x \to -\infty} f(x) = -\infty$

(minimum homework: 1.1 1-11 odds, 15, 19 and 21)

#15-26: Complete the table(s) and find the requested limits.

15) f(x) = 3x + 5, find

a) $\lim_{x \to 2-} (3x + 5)$

x	1.5	1.9	1.99	1.999
f(x)				

b) $\lim_{x \to 2+} (3x + 5)$

x	2.5	2.1	2.01	2.001
f(x)				

c) Use the results from part a and b to find: $\lim_{x \to 2} (3x + 5)$

(minimum homework: 1.1 1-11 odds, 15, 19 and 21)

16) f(x) = 2x - 3

a) $\lim_{x \to 4-} (2x - 3)$

x	3.5	3.9	3.99	3.999
f(x)				

b) $\lim_{x \to 4^+} f(x)$

x	4.5	4.1	4.01	4.001
f(x)				

c) Use the results from part a and b to find: $\lim_{x \to 4} f(x)$

Answer:

a) $\lim_{x \to 4-} (2\text{x} - 3)$

x	3.5	3.9	3.99	3.999
f(x)	4	4.8	4.98	4.998

b) $\lim_{x \to 4^+} f(x)$

x	4.5	4.1	4.01	4.001
f(x)	6	5.2	5.02	5.002

c) Use the results from part a and b to find: $\lim_{x \to 4} f(x) = 5$

(minimum homework: 1.1 1-11 odds, 15, 19 and 21)

17) f(x) = $\frac{x+2}{x-1}$ find

a) $\lim_{x \to 2-} \frac{x+2}{x-1}$

x	1.5	1.9	1.99	1.999
f(x)				

b) $\lim_{x \to 2+} \frac{x+2}{x-1}$

x	2.5	2.1	2.01	2.001
f(x)				

c) Use the results from part a and b to find: $\lim_{x \to 2} \frac{x+2}{x-1}$

(minimum homework: 1.1 1-11 odds, 15, 19 and 21)

18) $f(x) = \frac{x+5}{x+2}$

a) $\lim_{x \to 1-} \frac{x+5}{x+2}$

x	.5	.9	.99	.999
f(x)				

b) $\lim_{x \to 1^+} \frac{x+5}{x+2}$

x	1.5	1.1	1.01	1.001
f(x)				

c) Use the results from part a and b to find: $\lim_{x \to 1} \frac{x+5}{x+2}$

Answer:

a) $\lim_{x \to 1-} \frac{x+5}{x+2}$

x	.5	.9	.99	.999
f(x)	2.2	2.0344828	2.0033445	2.0003334

b) $\lim_{x \to 1^+} \frac{x+5}{x+2}$

x	1.5	1.1	1.01	1.001
f(x)	1.8571429	1.9677419	1.9966777	1.9996668

c) Use the results from part a and b to find: $\lim_{x \to 1} \frac{x+5}{x+2} = 2$

(minimum homework: 1.1 1-11 odds, 15, 19 and 21)

19) f(x) = $\frac{\sqrt{x}-3}{x-9}$, find

a) $\lim_{x \to 9-} \frac{\sqrt{x}-3}{x-9}$

x	8.5	8.9	8.99	8.999
f(x)				

b) $\lim_{x \to 9+} \frac{\sqrt{x}-3}{x-9}$

x	9.5	9.1	9.01	9.001
f(x)				

c) Use the results from part a and b to find: $\lim_{x \to 9} \frac{\sqrt{x}-3}{x-9}$

20) f(x) = $\frac{\sqrt{x}-2}{x-4}$

a) $\lim_{x \to 4-} \frac{\sqrt{x}-2}{x-4}$

x	3.5	3.9	3.99	3.999
f(x)				

b) $\lim_{x \to 4^+} \frac{\sqrt{x}-2}{x-4}$

x	4.5	4.1	4.01	4.001
f(x)				

c) Use the results from part a and b to find: $\lim_{x \to 4} \frac{\sqrt{x}-2}{x-4}$

Answer:

a) $\lim_{x \to 4-} \frac{\sqrt{x}-2}{x-4}$

x	3.5	3.9	3.99	3.999
f(x)	.25834261	.25158234	.25015645	.25001563

b) $\lim_{x \to 4^+} \frac{\sqrt{x}-2}{x-4}$

x	4.5	4.1	4.01	4.001
f(x)	.24264069	.24845673	.24984395	.24998438

c) Use the results from part a and b to find: $\lim_{x \to 4} \frac{\sqrt{x}-2}{x-4}$ 0.25

(minimum homework: 1.1 1-11 odds, 15, 19 and 21)

21) f(x) = $\frac{2x^2+3x+5}{x^2+4x-5}$

Complete the table to estimate $\lim_{x\to\infty} \frac{2x^2+3x+5}{x^2+4x-5}$

x	100	1000	100,000	1,000,000
f(x)				

22) f(x) = $\frac{6x^2+2x+5}{3x^2+4x-4}$

Complete the table to estimate $\lim_{x\to\infty} \frac{6x^2+2x+5}{3x^2+4x-4}$,

x	100	1000	100,000	1,000,000
f(x)				

Answer:

Complete the table to estimate $\lim_{x\to\infty} \frac{6x^2+2x+5}{3x^2+4x-4} = 2$

x	100	1000	100,000	1,000,000
f(x)	1.9806882	1.998007	1.99998	1.999998

(minimum homework: 1.1 1-11 odds, 15, 19 and 21)

23) f(x) = $\frac{6x^3-x^2+2x+5}{3x^4+4x^2-5x}$

Complete the table to estimate $\lim_{x\to\infty} \frac{6x^3-x^2+2x+5}{3x^4+4x^2-5x}$

x	100	1000	100,000	1,000,000
f(x)				

24) f(x) = $\frac{2x^2+2x-5}{3x^4-5x+2}$

Complete the table to estimate $\lim_{x\to\infty} \frac{6x^3-x^2+2x+5}{3x^4+4x^2-5x}$

x	100	1000	100,000
f(x)			

Answer:

24) f(x) = $\frac{2x^2+2x-5}{3x^4-5x+2}$

Complete the table to estimate $\lim_{x\to\infty} \frac{2x^2+2x-5}{3x^4-5x+2} = 0$

x	100	1000	100,000
f(x)	.00006731	.00000066733	.0000000000667

(minimum homework: 1.1 1-11 odds, 15, 19 and 21)

25) f(x) = $\frac{6x^5-x^2+2x+5}{3x^4+4x^2-5x}$

Complete the table to estimate $\lim_{x\to\infty} \frac{6x^5-x^2+2x+5}{3x^4+4x^2-5x}$

x	100	1000	10000	
f(x)				

26) f(x) = $\frac{2x^7+2x^3-5}{3x^4-5x}$

Complete the table to estimate $\lim_{x\to\infty} \frac{2x^7+2x^3-5}{3x^4-5x}$

x	100	1000	10000	
f(x)				

Answer:

Complete the table to estimate $\lim_{x\to\infty} \frac{2x^7+2x^3-5}{3x^4-5x} = \infty$

x	100	1000	10000	
f(x)	666667.78	666666667	666670000000000	